Determination of the inverse kinematics of a mechanic hand for realizing apprehension of a predetermined object with two or more fingers.

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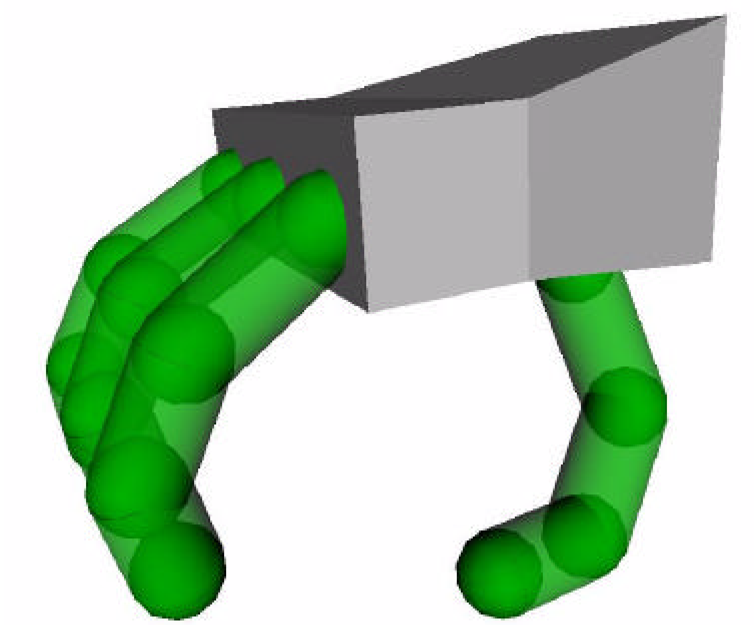

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PART 1

Introduction

The four fingered hand discussed in this work is based on the mechanical hand designed in the “Institut d´Organització i Control de Sistemes Industrials” (IOC), Barcelona, Spain. The hand consists of a rigid palm and the four fingers: thumb, index finger, middle finger and ring finger. It is meant to be similar in ratio compared to the anatomic hand of the human being. The most significant difference is the absence of the little finger. The proposed idea to grasp an object, given four contact points, can basically be applied directly to any mechanical hand which consists of the same number of articulations per finger.

The objective of this work is the determination of the configuration of a hand, first for two fingers and then extending to four fingers, which permits an “optimum“ grasp of a given object. The considered object is a parallelepiped defined by its width, height and a relatively big value for the length. The first two values are taken as input values to be able to define the location and the orientation of the contact points where the fingers make contact with the object. It also would be possible to define the contact points independently, without an explicit object. In this case, however, it would be essential to also specify the orientation of the surface at the contact points.

Once defined the contact points the hand will be placed around the object based on a heuristic method. This heuristic method does not necessarily define a possible solution with respect to the range of the articulations but defines a solution that is reachable for the fingers.

Based on the initial configuration of the hand a search is applied in order to improve and finally find the best configuration of the hand fulfilling the grasp. The search is guided by a target-function that calculates the quality of the determined configurations, favoring solutions in which the angles of the articulations remain close to the middle of their ranges and penalizes angles that are out of range and therefore would result in an impossible configuration of the hand. The calculated quality also has to take into account the height of the object to be able to guide the search in such a way that the configuration of the hand at least permits the desired height of the object.

The results basically can be divided into two types of solutions. In instances in which a small height is desired, the algorithm will find a configuration that is generous with respect to the height and would allow the object to intrude more into the found configuration of the hand. In other instances in which the desired height is relatively big, the search will be dominated by fulfilling the desired height and as a consequence the quality of the obtained configuration of the hand therefore will decrease rapidly, approaching more unusual configurations.

The proposed method for finding an appropriate configuration for the hand, first of all determines a solution for the middle finger together with the thumb and as a consequence the position of the wrist. In continuation, using the found configuration, the index finger and the ring finger are positioned afterwards. The work is based on the existing mechanical hand of the IOC which does not have a little finger. Nevertheless, the determination of the little finger could be solved in the same way as the last two fingers.

This document not only explains the theoretical part of the determination of the configuration of a four fingered mechanical hand but also shows results of its application in a simulation that is programed in the language C^{++}.

1.1 Nomenclature and Overview

In this section a simplified schematics of the hand (see Fig.1) together with the nomenclature (see Table-1) of the used variables can be found. Finally the sequence of the general proceeding (see Table-2) of the proposed method is shown.

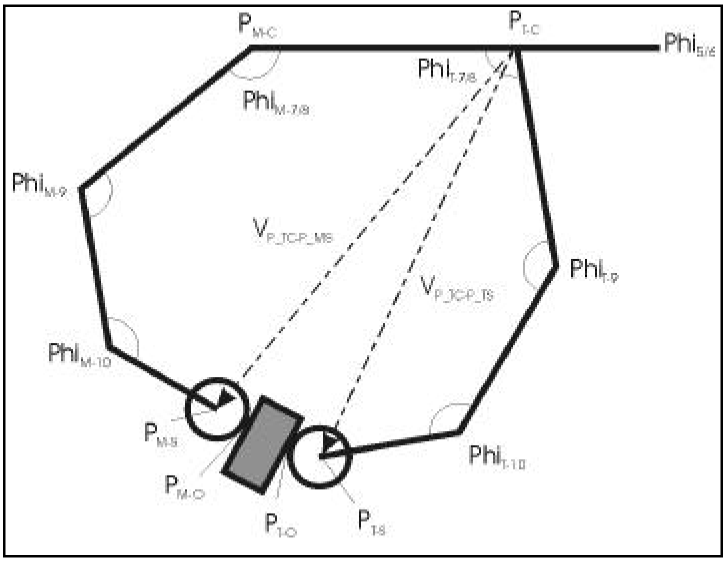


Figure 1. **Simplified schematic of hand showing palm, thumb and middle finger grasping the object.**

H = Hand	O = Contactpoint of Finger with Object	$D \in \Re$ = Distance
T = Thumb	S = Sphere in fingertip	$P \in \Re^3$ = Point
I = Index finger	C = Connection Finger-Hand	
M = Middle finger	Θ = Angle of articulation	
R = Ring finger		

Table 1. Nomenclature of used variables and indices.

1. Contact points:	
The contact point defines where the finger makes contact with the object.	P_{T-O}, P_{I-O}, P_{M-O}, P_{R-O}

2. Center of the spheres in the fingertips:	
The last phalanx of the finger terminates in a sphere. For the determination of the inverse kinematics the center of this sphere is taken into account. The center of the sphere can be determined by displacing the contact point perpendicular to the object´s surface with the length equal to the radius of the sphere.	P_{T-S}, P_{I-S}, P_{M-S}, P_{R-S}

3. Point of connection of thumb with palm with its heuristic method:	
The thumb makes contact with the palm in the point P_{T-C} where the base of the thumb is found (location of Θ_{T-7}). For initially positioning the Point P_{T-C} the following heuristic method is applied: Two spheres are positioned each surrounding one of the points P_{T-S} and P_{M-S}. Where the two spheres intersect, the initial position for P_{T-C} is located, which then will be improved by applying a guided search.	P_{T-C}

4. Determination of inverse kinematics of middle finger for Θ_{T-9} and Θ_{T-10} ($\Theta_{T-7} = 0$):	
a) Determination of a function for Θ_{M-8} over the distance between the points P_{T-C} and P_{T-S} fulfilling the target function.	Θ_{T-8}
b) Determination of the inverse kinematics of the middle finger for the angles Θ_{M-9}, Θ_{M-10}, depending on Θ_{M-8} and the distance of the points P_{T-C} and P_{M-S}.	Θ_{M-9}, Θ_{M-10}

5. Determination of the inverse kinematics of the thumb for Θ_{T-9} and Θ_{T-10}:	
Determination of the inverse kinematics of the thumb for the angles Θ_{T-9} and Θ_{T-10}, depending on the distance of the points P_{T-C} and P_{T-S}.	Θ_{T-9}, Θ_{T-10}

6. Determination of the angles Θ_{T-7} and Θ_{T-8}:	
With the determined values the middle finger and the thumb are positioned. The angles Θ_{T-7} and Θ_{T-8} define the orientatin of the thumb in relation to the the palm.	Θ_{T-7}, Θ_{T-8}

7. Determination of the angles Θ_{H-4}, Θ_{H-6}	
The found configuration permits the calculation of the angles Θ_{H-4} and Θ_{H-5} and the position of the hand-base P_{H-B}.	Θ_{H-4}, Θ_{H-6}, P_{H-B}

8. Determination of the angles Θ_{T-11}, Θ_{T-12}, Θ_{M-11}, Θ_{M-12} of the fingertips:	
With the vector pointing from the center of the sphere in the fingertip towards the point of contact with the object and the direction of the last phanlanx of the corresponding finger the angles Θ_{T-11}, Θ_{T-12}, Θ_{M-11} and Θ_{M-12} can be determined.	Θ_{T-11}, Θ_{T-12}, Θ_{M-11}, Θ_{M-12}

9. Determination of the articulations of the index finger and the ring finger:	
With the determination of the thumb, the palm and the middle finger it is possible to position the index finger and the ring finger.	Θ_{I-7} to Θ_{I-12}, Θ_{R-7} to Θ_{R-12}

10. Determination of contact points between the hand and the object:	
The found configuration of the hand allows the calculation of the position of the individual phalanges. With the position of the contact points of the parallelepiped it is then possible to evaluate the object´s maximal height.	Object-height

11. Application of a guided search for P_{T-C} to improve the configuration of the hand:	
For the articulations of the hand and for the height of the hand the quality can be calculated. This quality depends on the applied target function. In the case of the articulations a linear function is applied and in the case of the height of the object a quasi-binary function is applied. The guided search for the optimisatin of the configuration of the hand can be devided into two cases: • for relatively small values of the desired height of the object a configuration will be obtained that permits the object to intrude more into the hands configuration. • for relatively big values of the desired height of the object the search is dominated by fulfilling the desired height.	Final position of P_{T-C}

Table 2. Secuence of the general proceeding of the proposed method to determine the configuration of the hand grasping an object.

PART 2

Forward Kinematics

The work is based on the mechanical hand which exists in the IOC. The parameters of the hand are given as shown in the Tabel-3 to Tabel-6 in the Appendix, using the representation of Denavit-Hartenberg. Applying a set of angles for the articulations it is possible to calculate the final position of each finger, using these tables. This procedure is called **Forward Kinematics**.

Forward kinematics is the transformation from joint space to Cartesian space. This transformation depends on the configuration of the robot (i.e., link lengths, joint positions, type of each joint, etc.). In order to describe the location of each link relative to its neighbour, a frame is attached to each link, then a set of parameters is specified that characterize this frame. This notation is called the **Representation of Denavit-Hartenberg** which has become the standard way of representing robots and modelling their motions.

The method begins with a systematic approach to assigning and labelling an orthonormal coordinate system to each joint. It is then possible to relate one joint to the next and ultimately to assemble a complete representation of the geometry. Fig.-2 illustrates the parameters that connect the link i with the previous link i-1.

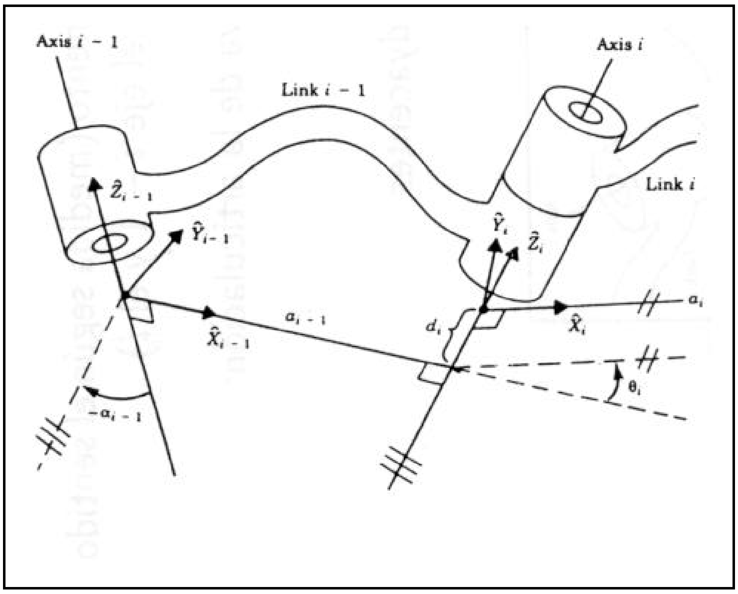


Figure 2. ***Four values (*** α_{i-1}, a_{i-1} , Θ_i, d_i***) can be identified relating the link i to the previous link i-1.***

The parameters used in the representation of Denavit-Hartenberg describe a set of 4x4 matrices:

A rotation α_{i-1} around the axis X_{i-1}:

$$ {}^{i-1}_{R}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_{i-1}) & -\sin(\alpha_{i-1}) & 0 \\ 0 & \sin(\alpha_{i-1}) & \cos(\alpha_{i-1}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

A translation a_{i-1} along the axis X_{i-1}:

$$ {}^{R}_{Q}T = \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

A rotation Θ_i around the axis Z_i:

$$ {}^{Q}_{P}T = \begin{bmatrix} \cos(\Theta_i) & -\sin(\Theta_i) & 0 & 0 \\ \sin(\Theta_i) & \cos(\Theta_i) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

A translation d_i along the axis Z_i:

$$ {}^{P}_{i}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

It can be noted that any of these values can be, and often are, equal to zero. Together, these four transformations in the above order lead to a unique homogeneous transformation matrix with four variables representing the relationship between these two links i and i-1.

The transformation matrix which gives the tranformation from link i-1 to link i is the following:

$$ {}^{i-1}_{i}T = \left({}^{i-1}_{R}T \cdot {}^{R}_{Q}T \cdot {}^{Q}_{P}T \cdot {}^{P}_{i}T \right) $$

PART 3 Definition of the object to be grasped.

Given a mechanical hand, the determination of its configuration starts with the calculation of the contact points of the object. These contact points then fulfill certain criteria, e.g. minimization of applied forces. These contact points are to be reached by the mechanical hand. However, some algorithms may determine contact points which are out of reach due to the restrictions of the kinematics of the given hand.

The objective of this work is not to find the best grasp configuration for a given object but to determine a good configuration of the hand for a given set of contact points. For simplification of the demonstration of the results the contact points for the simulation are calculated with the basic shape of a parallelepiped and are positioned in a single plane, the middle finger located exactly on the other side of the object where the thumb is positioned. The index finger and the ring finger then are positioned with a fixed distance next to the middle finger. The proposed method also would work for arbitrary contact points.

3.1 Definition of the object

The polyhedron utilized in the simulation is a parallelepiped, defined by its width, height and length (see Fig.3). The value of the length of the parallelepiped is relatively large and has no influence on the calculation. In other words, the parallelepiped to be grasped defines a rectangular bar. The width and the height of the object can take any arbitrary positive value. The object also can be displaced in the workspace which only results in a displacement of the hand and has no direct influence on the configuration of the hand.

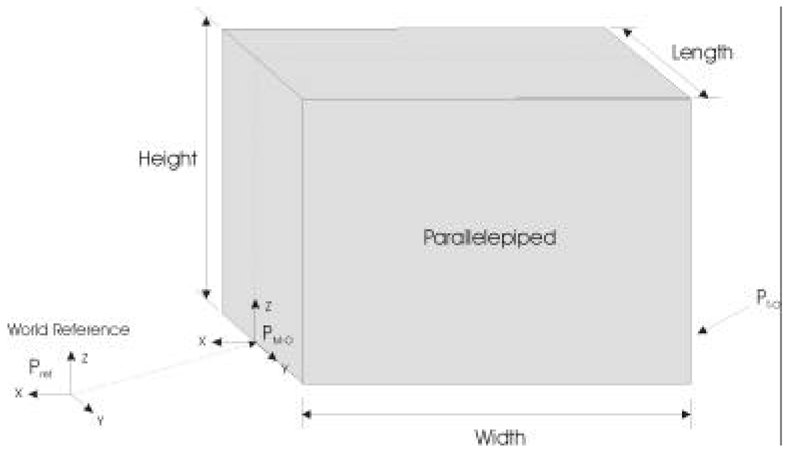


Figure 3. **Schematics of the parallelepiped defined by its length, height and width.**

PART 4 *The four fingered Hand*

4.1 Definition of the Hand

The schematics of the hand showing the base-vectors, necessary to calculate the thumb and the middle finger, can be found in the Appendix in Fig.13 and Fig.14. The reference point of the arrangement (hand with object) is defined by the coordinate system “World-Reference” with its three base vectors. Each finger has three rigid phalanges with an articulation inbetween ($\Theta_{Finger-9}$, $\Theta_{Finger-10}$), two more articulations can be found between the finger-base and the palm ($\Theta_{Finger-7}$, $\Theta_{Finger-8}$). The phalanges as well as the palm are rigid objects, hence their dimensions are constant values. A change in the configuration of the hand results from a change in the value of an angle of an articulation or in a displacement of the entire hand in one or more of the three directions of the base vectors. The description of the representation of Denavit-Hartenberg (see Section-2**)** explains how to understand the definition of the hand shown in Tabel-3 to Tabel-6 in the Appendix.

The objective of the work is to find an adequate configuration of the hand with respect to the freedom of movement of the object within the hand, rather than finding an optimal grasping configuration with respect to the object. In other words, the proposed method will find a configuration of the hand that permits to translate or rotate the object only with the fingers without changing the position of the palm. This is achieved by applying a target function that favors solutions in which the angles of the articulations remain close to the middle of their range and penalizes angles which are out of range and therefore would result in an impossible configuration of the hand.

4.1.1 Center of Sphere in Fingertip $P_{Finger-S}$

Each finger terminates in a fingertip which is formed by a sphere (see Fig.4). The radius of the sphere is defined by the value a_{13-1} of the respective finger. Once found the contact point with the object $P_{Finger-O}$, the center of the sphere of the fingertip $P_{Finger-S}$ can be determined. It can be calculated by displacing the contact point perpendicular to the object´s surface with the value of the radius of the sphere. This displacement is described by the vector $V_{Finger-OS}$. The equation for determining the position of the Center of the sphere is the following:

Thumb: $P_{T-S} = P_{T-O} - V_{T-OS}$

Index finger: $P_{I-S} = P_{I-O} - V_{I-OS}$

Middle finger: $P_{M-S} = P_{M-O} - V_{M-OS}$
Ring finger: $P_{R-S} = P_{R-O} - V_{R-OS}$

The vector $V_{Finger\text{-}OS}$ depends on the orientation of the surface of the object where the finger makes contact. In the case of the parallelepiped, which is oriented in the direction of the base-vectors, the vector points in the X-direction of the base vector of the "World-Reference". The vectors for the case of the parallelepiped are the following:

Thumb: $V_{T-OS} = \begin{bmatrix} -a_{13-1} \\ 0 \\ 0 \end{bmatrix}$ Index finger: $V_{I-OS} = \begin{bmatrix} a_{13-1} \\ 0 \\ 0 \end{bmatrix}$ Middle finger: $V_{M-OS} = \begin{bmatrix} a_{13-1} \\ 0 \\ 0 \end{bmatrix}$ Ring finger: $V_{R-OS} = \begin{bmatrix} a_{13-1} \\ 0 \\ 0 \end{bmatrix}$

In the sphere of the fingertip two virtual articulations ($\Theta_{Finger\text{-}11}$, $\Theta_{Finger\text{-}12}$) are considered (see Fig.13 and Fig.14), their determination is explained in Subsection-4.1.7.

In this work the object is set to a fixed orientation and can be translated in the direction of the three base vectors. A rotation of the object would result in a rotation of the complete arrangement and not in a variation of the angles of the fingers and therefore could be done after the determination of the configuration of the hand is completed.

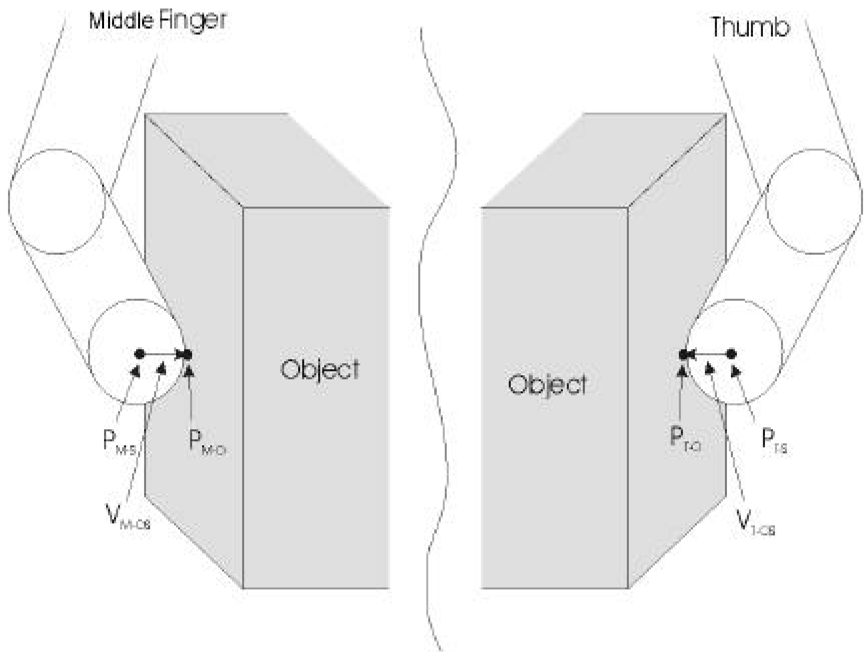


Figure 4. **Schematics of the sphere in the fingertips of the middle finger and the thumb, defined by the vectors $V_{M\text{-}OS}$ and $V_{T\text{-}OS}$ with its lengths equal to the radius of the spheres.**

4.1.2 Point of connection of thumb with palm $P_{T\text{-}C}$

The proposed method first determines a solution for the thumb and the middle finger and afterwards the positions of the remaining two fingers. The determination of the middle finger and the thumb can be devided into two parts. The determination of the thumb and the determination of the middle finger together with the palm. These two parts then have a common point $P_{T\text{-}C}$ which defines the connection of the thumb with the palm (see Fig.1 on page3).

The orientation of the thumb in $P_{T\text{-}C}$ in relation to the palm is defined by the articulations $\Theta_{T\text{-}7}$ and $\Theta_{T\text{-}8}$. Moving the hand always will result in a constant position of the point P_{T-C} within the palm which can be calculated using the forward kinematics of the thumb until its base system-7. Locating the base of the hand in the “World-reference” and setting the angles to its zero-position, the fixed location of $P_{T\text{-}C}$ within the hand can be found for the hand of the IOC:

$$P_{T-C} = \begin{bmatrix} 40{,}165 \\ 30{,}0122 \\ 145{,}814 \end{bmatrix}$$

In order to give a better understanding of where the thumb is located the following table shows the coordinates of the connection points of the other fingers:

Index finger	Middle finger	Ring finger
$P_{I-C} = \begin{bmatrix} 9.5 \\ 67 \\ 276{,}55 \end{bmatrix}$	$P_{M-C} = \begin{bmatrix} 9.5 \\ 0 \\ 276{,}55 \end{bmatrix}$	$P_{R-C} = \begin{bmatrix} 9.5 \\ -67 \\ 276{,}55 \end{bmatrix}$

It can be seen from the values in Z-direction that the thumb is connected to the palm below the other fingers. Besides it is displaced in X-direction, similar to the anatomic hand.

4.1.3 Heuristic method for determining the initial point $P_{T\text{-}C}$

The point $P_{T\text{-}C}$ serves as an auxiliary reference-point of the hand. Given the point $P_{T\text{-}C}$, a starting configuration for the palm and the fingers will be determined, depending on the distance between $P_{T\text{-}C}$ and the Center of the fingertips ($P_{T\text{-}S}$ and $P_{M\text{-}S}$).

The objective of applying this heuristic method is to locate $P_{T\text{-}C}$ such that it permits the hand to grasp the object. In other words this heuristic method ensures that the contact points are in reach of the fingers. The second objective is to locate the hand in a good starting position to keep the computing time for the following search small.

The heuristic method proposes to surround each of the two points P_{T-S} and P_{M-S} by a circle, each having a predetermined radius. For simplification the thumb is located opposite to the middle finger, exactly on the other side of the object. The circles therefore are formed in a plane parallel to the plane containing the X- and Z-base-vectors of the World-Reference. The intersection of the two circles which is located above the object is the initial point P_{T-C} for a given distance between the points P_{T-S} and P_{M-S}.

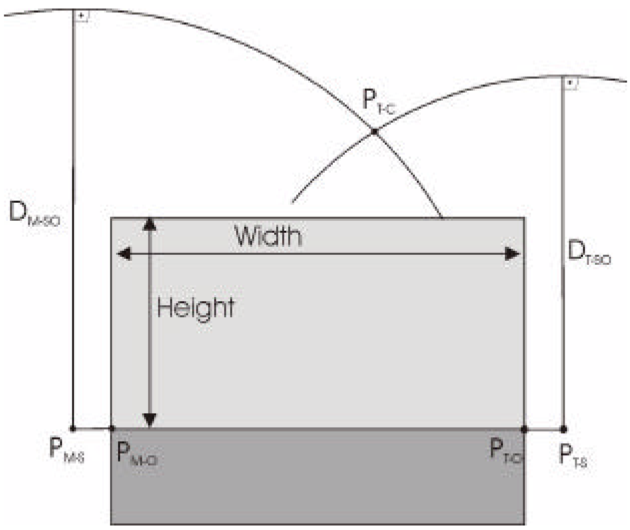


Figure 5. Schematic of object with surrounding circles.

For objects with a relatively big value for its width the final values of the articulations of the hand will result in a more stretched configuration, small values will permit the articulations to stay close to the middle of their range as it is forced by the target function. Therefore the radius of the two circles vary depending on the distance between the points P_{T-S} and P_{M-S}. The thumb and the middle finger have a maximal distance of reach (MAX), a distance of reach when the values of the articulations are in the middle of their range (MID) and a minimal distance of reach (MIN). The distances are defined by P_{T-C}, P_{T-S} and P_{T-C}, P_{M-S} respectively.

The distances for the mechanical hand are as follows:

Thumb:	Middle finger:
$MIN\{D_{T-SO}\} = 75,90$	$MIN\{D_{M-SO}\} = 81,45$
$MID\{D_{T-SO}\} = 150,26$	$MID\{D_{M-SO}\} = 193,78$
$MAX\{D_{T-SO}\} = 181,83$	$MAX\{D_{M-SO}\} = 300,10$

The maximal theoretical distance then would be ($MAX\{D_{T-SO}\}+MAX\{D_{M-SO}\}$). This is only a theoretical value and because of the limitation of the angles Θ_{T-7} and Θ_{T-8}. It is not possible to strech both fingers and let them point in oposite directions. However, this theoretical value is taken into account in the determination of the initial point P_{T-S}.

For the width of the object being nearly zero, the distance between the points $P_{T\text{-}S}$ and $P_{M\text{-}S}$ is calculated by $(a_{T13-1} + a_{M13-1})$ which is the radius of the spheres in the fingertips of the thumb and the middle finger, respectively . In the case of the mechanical hand of the IOC this would result in a distance of 40mm which is less then the difference of ($MID\{D_{M\text{-}SO}\}$-$MID\{D_{T\text{-}SO}\}$). Therefore in the calculation of the initial position of $P_{T\text{-}C}$ instead of the value $MID\{D_{M\text{-}SO}\}$, the value $MID\{D_{T\text{-}SO}\}$ is used for both fingers.

The coordinates of the point $P_{T\text{-}C}$ are determined using the auxiliary variables a,b, c with $D_{TS\text{-}MS}$ being the distance between the points $P_{T\text{-}S}$ and $P_{M\text{-}S}$ (see Fig.6).

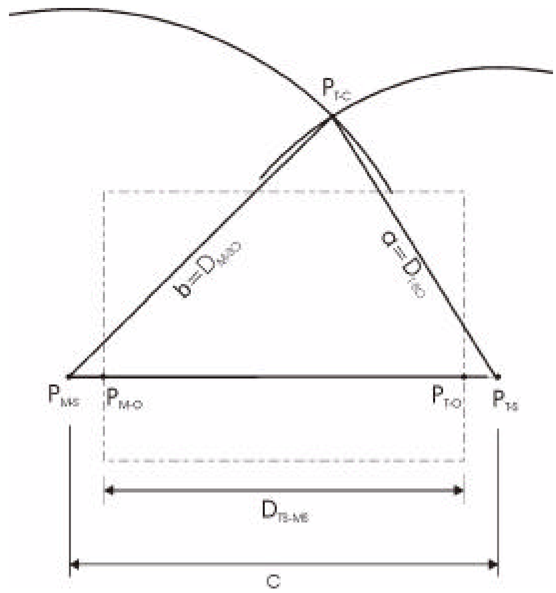


Figure 6. Schematics showing auxiliary distances.

The value a is the variable that indicates the distance between the center of sphere of the thumb with the point $P_{T\text{-}C}$. The interval of a is $MID\{D_{T\text{-}SO}\}$ to $MAX\{D_{T\text{-}SO}\}$.
The value b is the variable that indicates the distance between the center of sphereof the middle finger with the point $P_{T\text{-}C}$. The interval of b is $MID\{D_{T\text{-}SO}\}$ to $MAX\{D_{M\text{-}SO}\}$.
The value c is the variable that indicates the distance between the center spheres of the thumb and the middle finger. The interval of c is $(a_{T13-1} + a_{M13-1})$ to $(a_{T13-1} + a_{M13-1} + D_{TS\text{-}MS})$.

The values for a, b and c are calculated using the follwoing formulas:

- $a = D_{T\text{-}SO} = MID(D_{T\text{-}SO}) + (MAX(D_{T\text{-}SO}) - MID(D_{T\text{-}SO})) \cdot \dfrac{D_{TS\text{-}MS}}{MAX(D_{T\text{-}SO}) + MAX(D_{M\text{-}SO})}$
- $b = D_{M\text{-}SO} = MID(D_{T\text{-}SO}) + (MAX(D_{T\text{-}SO}) - MID(D_{T\text{-}SO})) \cdot \dfrac{D_{TS\text{-}MS}}{MAX(D_{T\text{-}SO}) + MAX(D_{M\text{-}SO})}$
- $c = (a_{T13-1} + a_{M13-1}) + D_{TS\text{-}MS}$

The point $P_{T\text{-}C}$ then can be calculated using the law of cosine:

- $P_{T\text{-}C}(X) = P_{M\text{-}O}(X) + \frac{-a^2 + b^2 + c^2}{2 \cdot c}$
- $P_{T\text{-}C}(Y) = P_{M\text{-}O}(Y)$
- $P_{T\text{-}C}(Z) = P_{M\text{-}O}(Z) + \sqrt{b^2 + -P_{T\text{-}C}(x)^2}$

The above equations calculate the intersection between the two circles which in this case are formed in one and the same plane with the points of contact with the object.

4.1.4 Inverse kinematics of middle finger

The middle finger consists of the two articulations $\Theta_{M\text{-}9}$ and $\Theta_{M\text{-}10}$ and the two articulations $\Theta_{M\text{-}7}$ and $\Theta_{M\text{-}8}$ form the connection between the base of the middle finger and the palm. The articulation $\Theta_{M\text{-}7}$ describes the rotation of the finger sidewards (abduction). In the proposed method the angle $\Theta_{M\text{-}7}$ is fixed to zero.

Restriction for the middle finger: $\mathbf{\Theta_{M\text{-}7} = 0}$

This restriction will only be applied for the middle finger. For the thumb it is necessary to maintain the possibility to rotate $\Theta_{T\text{-}7}$ in order to reach possible configurations. Later on the angles $\Theta_{I\text{-}7}$ and $\Theta_{R\text{-}7}$ for the index finger and the ring finger, respectively, also will be necessary to determine in order to be able to position the last two fingers.

The angles $\Theta_{M\text{-}8}$, $\Theta_{M\text{-}9}$ and $\Theta_{M\text{-}10}$ form the connection between the two points $P_{T\text{-}C}$ and $P_{M\text{-}S}$. These three angles can be found in one plane and therefore permit infinite solutions. In order to be able to solve this problem the angle $\Theta_{M\text{-}8}$ needs to be determined in advance. The target-function applied in the search favors solutions which result in angles remaining close to the middle of their range.

The location of the sphere in the fingertip for the middle finger, in relation to the base of the middle finger can be found using the forward kinematics. This leeds to the following values:

$$\begin{aligned} X = {} & 33.62 \cdot \cos(\Theta_{M\text{-}10}) \cdot \sin(\Theta_{M\text{-}8}) \cdot \cos(\Theta_{M\text{-}9}) + 33.62 \cdot \cos(\Theta_{M\text{-}10}) \cdot \cos(\Theta_{M\text{-}8}) \cdot \sin(\Theta_{M\text{-}9}) \\ & + 33.62 \cdot \sin(\Theta_{M\text{-}10}) \cdot \cos(\Theta_{M\text{-}8}) \cdot \cos(\Theta_{M\text{-}9}) - 33.62 \cdot \sin(\Theta_{M\text{-}10}) \cdot \sin(\Theta_{M\text{-}8}) \cdot \sin(\Theta_{M\text{-}9}) \\ & + 56 \cdot \sin(\Theta_{M\text{-}8}) \cdot \cos(\Theta_{M\text{-}9}) + 56 \cdot \cos(\Theta_{M\text{-}8}) \cdot \sin(\Theta_{M\text{-}9}) + 76.66 \cdot \sin(\Theta_{M\text{-}8}) + 9.5 \end{aligned}$$

$$Y = 0$$

$$\begin{aligned} Z = {} & 33.62 \cdot \cos(\Theta_{M\text{-}10}) \cdot \cos(\Theta_{M\text{-}8}) \cdot \cos(\Theta_{M\text{-}9}) - 33.62 \cdot \cos(\Theta_{M\text{-}10}) \cdot \sin(\Theta_{M\text{-}8}) \cdot \sin(\Theta_{M\text{-}9}) \\ & - 33.62 \cdot \sin(\Theta_{M\text{-}10}) \cdot \cos(\Theta_{M\text{-}8}) \cdot \sin(\Theta_{M\text{-}9}) - 33.62 \cdot \sin(\Theta_{M\text{-}10}) \cdot \sin(\Theta_{M\text{-}8}) \cdot \cos(\Theta_{M\text{-}9}) \\ & + 56 \cdot \cos(\Theta_{M\text{-}8}) \cdot \cos(\Theta_{M\text{-}9}) - 56 \cdot \sin(\Theta_{M\text{-}8}) \cdot \sin(\Theta_{M\text{-}9}) + 76.66 \cdot \cos(\Theta_{M\text{-}8}) + 276.55 \end{aligned}$$

The calculation of the euclidean distance between the points $P_{T\text{-}C}$ and $P_{M\text{-}S}$, substituting $\Theta_{M\text{-}9}$ and $\Theta_{M\text{-}10}$ for $\Theta_{M\text{-}8}$, results in the following equation:

$$D^2_{TC\text{-}MS} = 100 \cdot (-0{,}8248 \cdot \cos(\Theta_{M-8})^2 \cdot \sin(\Theta_{M-8}) - 0{,}687 \cdot \cos(\Theta_{M-8}) \cdot \sin(\Theta_{M-8}) + 3{,}96 \cdot \cos(\Theta_{M-8})^2 + 3{,}516 \cdot \cos(\Theta_{M-8}) - 0{,}264 \cdot \sin(\Theta_{M-8}) + 0{,}602 \cdot \cos(\Theta_{M-8}) + 0{,}9279)$$

The calculation of the inverse of this function in order to obtain the function of $\Theta_{M\text{-}8}$ depending on the distance is solved numerically with a polynomial function of fifth degree which follows the original function with a maximal error of 0,6º. The determined function for $\Theta_{M\text{-}8}$ is the following:

$$\Theta_{M-8} = -0{,}002 \cdot D^5 + 0{,}0676 \cdot D^4 - 0{,}8896 \cdot D^3 + 5{,}6392 \cdot D^2 - 22{,}144 \cdot D + 100{,}48$$

With ($D = (D_{TC\text{-}MS} - 84)/18 + 1$) and the variable $D_{TC\text{-}MS}$ being the distance between the points $P_{T\text{-}C}$ and $P_{M\text{-}S}$.

As a second step it is necessary to determine the position of the resulting endpoint of the finger in order to determine the exact value for the angles $\Theta_{M\text{-}9}$ and $\Theta_{M\text{-}10}$. It is possible to calculate the approximate X-value of the endpoint with the forward kinematics using the calculated angle $\Theta_{M\text{-}8}$ ($\Theta_{M\text{-}8}$ has a maximal error of 0,6º).

The Y-value is a constant value of 0 due to the fact that the angle $\Theta_{M\text{-}7}$ is set to 0º and therefore the endpoint of the middle finger moves in a fixed plane. The Z-value then is calculated with the Euclidean-distance of the points $P_{T\text{-}C}$ to $P_{M\text{-}S}$ which was the basis for calculating the angle $\Theta_{M\text{-}8}$:

$$X = 134{,}48 \cdot \cos(\Theta_{M\text{-}8})^3 - 24{,}2 \cdot \cos(\Theta_{M\text{-}8}) + 112 \cdot \cos(\Theta_{M\text{-}8})^2 - 56$$
$$Y = 0$$
$$Z = \sqrt{D_{TC\text{-}MS}{}^2 - X^2}$$

Once determined the articulation $\Theta_{M\text{-}8}$ and the endpoint of the finger it is possible to solve the inverse kinematics for $\Theta_{M\text{-}9}$ and $\Theta_{M\text{-}10}$ of the middle finger.

The direct kinematics of center of the sphere of the middle finger is calculated using the following matrices:

$$^{4}_{11}T_{M\text{-}ges} = {}^{4}_{5}T_M \cdot {}^{5}_{6}T_M \cdot {}^{6}_{7}T_M \cdot {}^{7}_{8}T_M \cdot {}^{8}_{9}T_M \cdot {}^{9}_{10}T_M \cdot {}^{10}_{11}T_M$$

The matrix $^{4}_{11}T_{M\text{-}ges}$ contains two types of information. The first is the 3x3 matrix which describes the orientation of the center of the sphere and the 3x1 matrix which describes its position with the coordinates X, Y and Z. The previously calculated position of the center of the sphere is calculated in reference to the hand. Therefore the angles $\Theta_{H\text{-}4}$, $\Theta_{H\text{-}5}$, $\Theta_{H\text{-}6}$, $\Theta_{M\text{-}7}$ and $\Theta_{M\text{-}11}$ can be set to cero.

The X and Z position of the center of the sphere of the middle finger then is determined with the forward kinematics:

$$\begin{aligned}\mathbf{X} = \; & 33{,}62 \cdot \sin(\Theta_{M\text{-}8}) \cdot \cos(\Theta_{M\text{-}9}) \cdot \cos(\Theta_{M\text{-}10}) + 33{,}62 \cdot \cos(\Theta_{M\text{-}8}) \cdot \sin(\Theta_{M\text{-}9}) \cdot \cos(\Theta_{M\text{-}10}) - \\ & 33{,}62 \cdot \sin(\Theta_{M\text{-}8}) \cdot \sin(\Theta_{M\text{-}9}) \cdot \sin(\Theta_{M\text{-}10}) + 33{,}62 \cdot \cos(\Theta_{M\text{-}8}) \cdot \cos(\Theta_{M\text{-}9}) \cdot \sin(\Theta_{M\text{-}10}) + \\ & 56 \cdot \sin(\Theta_{M\text{-}8}) \cdot \cos(\Theta_{M\text{-}9}) + 56 \cdot \cos(\Theta_{M\text{-}8}) \cdot \sin(\Theta_{M\text{-}9}) + 76{,}66 \cdot \sin(\Theta_{M\text{-}8}) + 9{,}5\end{aligned}$$

$$\mathbf{Y} = 0$$

$$\begin{aligned}\mathbf{Z} = \; & -33{,}62 \cdot \sin(\Theta_{M\text{-}8}) \cdot \sin(\Theta_{M\text{-}9}) \cdot \cos(\Theta_{M\text{-}10}) + 33{,}62 \cdot \cos(\Theta_{M\text{-}8}) \cdot \cos(\Theta_{M\text{-}9}) \cdot \cos(\Theta_{M\text{-}10}) - \\ & 33{,}62 \cdot \cos(\Theta_{M\text{-}8}) \cdot \sin(\Theta_{M\text{-}9}) \cdot \sin(\Theta_{M\text{-}10}) - 33{,}62 \cdot \sin(\Theta)_{M\text{-}8} \cdot \cos(\Theta_{M\text{-}9}) \cdot \sin(\Theta_{M\text{-}10}) + \\ & 56 \cdot \cos(\Theta_{M\text{-}8}) \cdot \cos(\Theta_{M\text{-}9}) + 56 \cdot \sin(\Theta_{M\text{-}8}) \cdot \sin(\Theta_{M\text{-}9}) + 76{,}66 \cdot \cos(\Theta_{M\text{-}8}) + 276{,}55\end{aligned}$$

In the following the two basic trigonometric transformations are used:

$$\sin(x_1 \pm x_2) = \sin(x_1) \cdot \cos(x_2) \pm \cos(x_1) \cdot \sin(x_2)$$
$$\cos(x_1 \pm x_2) = \cos(x_1) \cdot \cos(x_2) \mp \sin(x_1) \cdot \sin(x_2)$$

This leeds to the following simplification:

$$\mathbf{X} = 33{,}62 \cdot \sin(\Theta_{M\text{-}8} + \Theta_{M\text{-}9} + \Theta_{M\text{-}10}) + 56 \cdot \sin(\Theta_{M\text{-}8} + \Theta_{M\text{-}9}) + 76{,}66 \cdot \sin(\Theta_{M\text{-}8}) + 9{,}5$$

$$\mathbf{Z} = 33{,}62 \cdot \cos(\Theta_{M\text{-}8} + \Theta_{M\text{-}9} + \Theta_{M\text{-}10}) + 56 \cdot \cos(\Theta_{M\text{-}8} + \Theta_{M\text{-}9}) + 76{,}66 \cdot \cos(\Theta_{M\text{-}8}) + 276{,}55$$

Solving for ($\sin(\Theta_{M\text{-}8} + \Theta_{M\text{-}9} + \Theta_{M\text{-}10})$) and ($\cos(\Theta_{M\text{-}8} + \Theta_{M\text{-}9} + \Theta_{M\text{-}10})$), respectively:

$$\sin(\Theta_{M\text{-}8} + \Theta_{M\text{-}9} + \Theta_{M\text{-}10}) = \frac{X - 56 \cdot \sin(\Theta_{M\text{-}8} + \Theta_{M\text{-}9}) - 76{,}66 \cdot \sin(\Theta_{M\text{-}8}) - 9{,}5}{33{,}62}$$

$$\cos(\Theta_{M\text{-}8} + \Theta_{M\text{-}9} + \Theta_{M\text{-}10}) = \frac{Z - 56 \cdot \cos(\Theta_{M\text{-}8} + \Theta_{M\text{-}9}) - 76{,}66 \cdot \cos(\Theta_{M\text{-}8}) - 276{,}55}{33{,}62}$$

Applying the fact that $\tan(x) = \frac{\sin(x)}{\cos(x)}$:

$$\tan(\Theta_{M\text{-}8} + \Theta_{M\text{-}9} + \Theta_{M\text{-}10}) = \frac{X - 56 \cdot \sin(\Theta_{M\text{-}8} + \Theta_{M\text{-}9}) - 76{,}66 \cdot \sin(\Theta_{M\text{-}8}) - 9{,}5}{Z - 56 \cdot \cos(\Theta_{M\text{-}8} + \Theta_{M\text{-}9}) - 76{,}66 \cdot \cos(\Theta_{M\text{-}8}) - 276{,}55}$$

The angle $\Theta_{M\text{-}10}$ of the middle finger can then be calculated:

$$\boxed{\Theta_{M\text{-}10} = \operatorname{atan}\left(\frac{(X - 56 \cdot \sin(\Theta_{M\text{-}8} + \Theta_{M\text{-}9}) - 76{,}66 \cdot \sin(\Theta_{M\text{-}8}) - 9{,}5)}{(Z - 56 \cdot \cos(\Theta_{M\text{-}8} + \Theta_{M\text{-}9}) - 76{,}66 \cdot \cos(\Theta_{M\text{-}8}) - 276{,}55)}\right) - \Theta_{M\text{-}8} - \Theta_{M\text{-}9}}$$

The angle $\Theta_{M\text{-}10}$ can be solved provided that the angle $\Theta_{M\text{-}9}$ is known. The angle $\Theta_{M\text{-}9}$ will be determined in the following subsection.

For the solution of the angle Θ_9 of the middle finger the trigonometric relation is used:

$$\cos(x)^2 + \sin(x)^2 = 1$$

therefore: $\cos(\Theta_{M\text{-}8} + \Theta_{M\text{-}9} + \Theta_{M\text{-}10})^2 + \sin(\Theta_{M\text{-}8} + \Theta_{M\text{-}9} + \Theta_{M\text{-}10})^2 = 1$

Using the previously obtained relation for $\cos(\Theta_{M\text{-}8} + \Theta_{M\text{-}9} + \Theta_{M\text{-}10})$ and $\sin(\Theta_{M\text{-}8} + \Theta_{M\text{-}9} + \Theta_{M\text{-}10})$:

$$\left(\frac{X - 56 \cdot \sin(\Theta_{M\text{-}8} + \Theta_{M\text{-}9}) - 76{,}66 \cdot \sin(\Theta_{M\text{-}8}) - 9{,}5}{33{,}62}\right)^2 + \left(\frac{Z - 56 \cdot \cos(\Theta_{M\text{-}8} + \Theta_{M\text{-}9}) - 76{,}66 \cdot \cos(\Theta_{M\text{-}8}) - 276{,}55}{33{,}62}\right)^2 = 1$$

This ecuation does not contain the angle $\Theta_{M\text{-}10}$ and the angle $\Theta_{M\text{-}8}$ is already predetermined. Solving this Ecuation leeds to the following result:

$$a_1 = 2 \cdot 9{,}5 \cdot 56 + 2 \cdot 56 \cdot 76{,}66 \cdot \sin(\Theta_{M\text{-}8}) - 2 \cdot 56 \cdot X$$

$$a_2 = 2 \cdot 276{,}55 \cdot 56 + 2 \cdot 56 \cdot 76{,}66 \cdot \cos(\Theta_{M\text{-}8}) - 2 \cdot 56 \cdot Z$$

$$a = \sqrt{a_1^2 + a_1^2}$$

$$k = Z^2 + X^2 - 2 \cdot 276{,}55 - 2 \cdot 9{,}5 \cdot X - 2 \cdot 76{,}66 \cdot (Z \cdot \cos(\Theta_{M\text{-}8}) + X \cdot \cos(\Theta_{M\text{-}8}))$$

$$2 \cdot 76{,}66 \cdot 276{,}55 \cdot \cos(\Theta_{M\text{-}8}) + 2 \cdot 76{,}66 \cdot 9{,}5 \cdot \sin(\Theta_{M\text{-}8}) + 84452{,}6037$$

$$\boxed{\Theta_{M\text{-}9} = \begin{cases} \pi - \operatorname{asin}\left(-\frac{k}{a}\right) - \Theta_{M\text{-}8} - \operatorname{atan}\left(\frac{a_2}{a_1}\right) & \text{if a1} \geq 0 \text{and a2} \geq 0 \\ -\pi - \operatorname{asin}\left(-\frac{k}{a}\right) - \Theta_{M\text{-}8} - \operatorname{atan}\left(\frac{a_2}{a_1}\right) & \text{if a1} \geq 0 \text{and a2} < 0 \\ -\operatorname{asin}\left(-\frac{k}{a}\right) - \Theta_{M\text{-}8} - \operatorname{atan}\left(\frac{a_2}{a_1}\right) & \text{if a1} < 0 \end{cases}}$$

With the obtained values it is possible to calculate the angles $\Theta_{H\text{-}4}$ and $\Theta_{H\text{-}6}$ which are located in the base of the hand. In Fig.7 a surrounding box for the middle finger is defined. The point P_0 indicates the point $P_{T\text{-}C}$ and the point P_4 indicates the center of the sphere of the middle finger. The values X_F, Y_F and Z_F determine the distance of the center of the sphere to the point $P_{T\text{-}C}$.

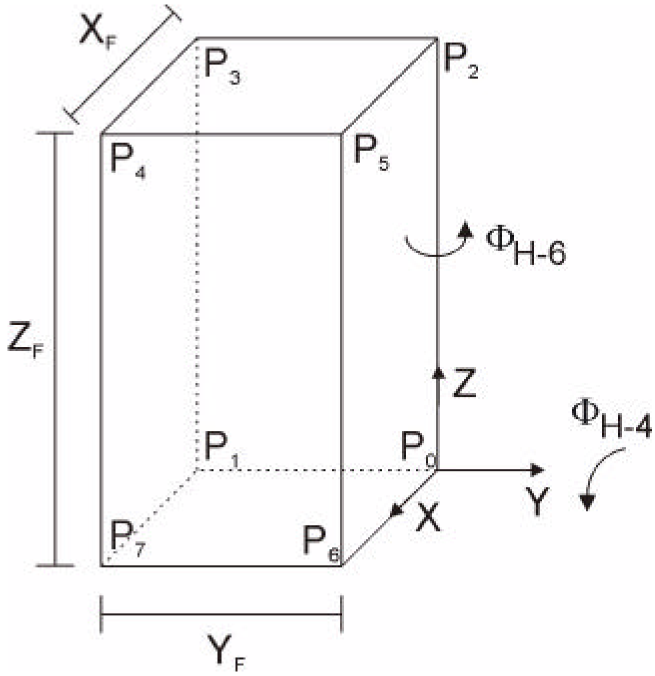


Figure 7. **Schematics for determinatino of angles Θ_{H-4} and Θ_{H-6}.**

In general the vector that points from P_0 to P_4 is the following:

$$P_4 = \begin{bmatrix} X_F \\ -Y_F \\ Z_F \end{bmatrix}$$

With the representation of Denavit-Hartenberg it is possible to define the following matrices:

Rotation in Z: $$M_Z = \begin{bmatrix} \cos(\Theta_{H-4}) & 0 & \sin(\Theta_{H-4}) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\Theta_{H-4}) & 0 & \cos(\Theta_{H-4}) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation in Y: $$M_Y = \begin{bmatrix} \cos(\Theta_{H-6}) & -\sin(\Theta_{H-6}) & 0 & 0 \\ \sin(\Theta_{H-6}) & \cos(\Theta_{H-6}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Adding the fourth dimension to the vector P_4 it is possible to rotate it around the Z-axis and around the Y-axis:

$$M_Y \cdot M_Z \cdot P_4 = \begin{bmatrix} X_F \cdot \cos(\Theta_{H-6}) \cdot \cos(\Theta_{H-5}) + Y_F \cdot \sin(\Theta_{H-6}) + Z_F \cdot \cos(\Theta_{H-6}) \cdot \sin(\Theta_{H-5}) \\ X_F \cdot \sin(\Theta_{H-6}) \cdot \cos(\Theta_{H-5}) - Y_F \cdot \cos(\Theta_{H-6}) + Z_F \cdot \sin(\Theta_{H-6}) \cdot \sin(\Theta_{H-5}) \\ -X_F \cdot \sin(\Theta_{H-5}) + Z_F \cdot \cos(\Theta_{H-5}) \\ 0 \end{bmatrix}$$

For simplification the following trigonometric rules are applied:

1. Conversion to Cos: $a_1 \cdot \sin(x) + a_2 \cdot \cos(x) = a \cdot \cos(x + \varphi)$

   with $a = \sqrt{a_1 + a_2}$ and $\varphi = \operatorname{atan}\left(\frac{a_1}{a_2}\right)$

2. Conversion to Sin: $a_1 \cdot \sin(x) + a_2 \cdot \cos(x) = a \cdot \sin(x + \varphi)$

   with $a = \sqrt{a_1 + a_2}$ and $\varphi = \operatorname{atan}\left(-\frac{a_1}{a_2}\right)$

The result for Θ_{H-5} then is the following with $P_{M-S}(z)$ and $P_{T-C}(z)$ being the values in z-direction of these points.

$$\boxed{\Theta_{H-5} = \operatorname{acos}\left(\frac{P_{M-S}(z) - P_{T-C}(z)}{\sqrt{Z_F^2 + X_F^2}}\right) - \operatorname{atan}\left(\frac{X_F}{Z_F}\right)}$$

The result for Θ_{H-6} then is the following:

$$\boxed{\Theta_{H-6} = \operatorname{asin}\left(\frac{(P_{M-S}(x) - P_{T-C}(x)) \cdot Y_F + (P_{M-S}(y) - P_{T-C}(y)) \cdot m}{m^2 + Y_F^2}\right)}$$

with $\boxed{m = (X_F \cdot \cos(\Theta_{H-5}) + Z_F \cdot \sin(\Theta_{H-5}))}$

For the hand of the IOC the constant value Y_F is equal to 30,0122 which is the displacement of the point P_{T-C} with respect to the base of the finger in y-direction.

4.1.5 Inverse kinematics of thumb

The thumb consists of the two articulations Θ_{T-9} and Θ_{T-10} and the two articulations Θ_{T-7} and Θ_{T-8} form the connection to the palm in the point P_{T-C}. In this subsection the angle Θ_{T-8} will be calculated without taking in account that the thumb is connected with the palm and in the following subsections the angle Θ_{T-8} and Θ_{T-7} will be calculated with respect to the orientation of the palm.

The three angles Θ_{T-8}, Θ_{T-9} and Θ_{T-10} can be found in one plane and therefore permit infinite solutions. In order to be able to solve this problem the angle Θ_{T-8} needs to be predetermined, similar to the middle finger. Therefore the same method applied for the middle finger is used for the thumb as well.

The calculation of the euclidean distance between the points P_{T-C} and P_{T-S}, substituting Θ_{T-9} and Θ_{T-10} for Θ_{T-8}, results in the following equation:

$$D^2_{TC-TS} = [156{,}68 \cdot \cos(\Theta_{T-8})^3 - 40{,}85 \cdot \cos(\Theta_{T-8}) + 132 \cdot \cos(\Theta_{T-8})^2 - 66]^2$$
$$0{,}0001 \cdot \sin(\Theta_{T-8})^2 \cdot (15668 \cdot \cos(\Theta_{T-8})^2 + 3749 + 13200 \cdot \cos(\Theta_{T-8})^2)^2$$

The calculation of the inverse of this function in order to obtain the function of Θ_{T-8} depending on the distance is solved numerically with a polynomial function of fifth degree. The determined function for Θ_{T-8} is the following:

$$\Theta_{T-8} = -0{,}018 \cdot D^5 + 0{,}341 \cdot D^4 - 2{,}4576 \cdot D^3 + 8{,}075 \cdot D^2 - 20{,}181 \cdot D + 104.18$$

With ($D = (D_{TC-TS} - 84)/18 + 1$) and the variable D_{TC-TS} being the distance between the points P_{T-C} and P_{T-S}.

As a second step it is necessary to determine the position of the resulting endpoint of the finger in order to determine the exact value for the angles Θ_{T-9} and Θ_{T-10}. It is possible to calculate the approximate X-value of the endpoint with the forward kinematics using the calculated angle Θ_{T-8}.

$$X = 156{,}68 \cdot \cos(\Theta_{T-8})^3 - 40{,}85 \cdot \cos(\Theta_{T-8}) + 132 \cdot \cos(\Theta_{T-8})^2 - 66$$
$$Y = 0$$
$$Z = \sqrt{D_{TC-TS}^2 - X^2}$$

Once determined the articulation Θ_{T-8} and the endpoint of the finger it is possible to solve the inverse kinematics for Θ_{T-9} and Θ_{T-10} of the thumb.

The direct kinematics of center of the sphere of the thumb from the with respect to the point $P_{T\text{-}C}$ is calculated using the following matrices:

$$ {}^{8}_{11}T_{T\text{-}ges} = {}^{8}_{9}T_{T} \cdot {}^{9}_{10}T_{T} \cdot {}^{10}_{11}T_{T} $$

As already applied for the middle finger the information taken into account is the position rather than the orientation that can be calculated. This information can be found in the 3x1 matrix describing the coordinates X, Y and Z.

The X and Z position of the center of the sphere of the thumb then is determined with the forward kinematics:

$$ \begin{aligned} \mathbf{X} = \; & 39{,}17 \cdot \cos(\Theta_{T\text{-}8}) \cdot \cos(\Theta_{T\text{-}9}) \cdot \cos(\Theta_{T\text{-}10}) + 39{,}17 \cdot \sin(\Theta_{T\text{-}8}) \cdot \sin(\Theta_{T\text{-}9}) \cdot \cos(\Theta_{T\text{-}10}) - \\ & 39{,}17 \cdot \cos(\Theta_{T\text{-}8}) \cdot \sin(\Theta_{T\text{-}9}) \cdot \sin(\Theta_{T\text{-}10}) - 39{,}17 \cdot \sin(\Theta_{T\text{-}8}) \cdot \cos(\Theta_{T\text{-}9}) \cdot \sin(\Theta_{T\text{-}10}) + \\ & 66 \cdot \cos(\Theta_{T\text{-}8}) \cdot \cos(\Theta_{T\text{-}9}) - 66 \cdot \sin(\Theta_{T\text{-}8}) \cdot \sin(\Theta_{T\text{-}9}) + 76{,}66 \cdot \cos(\Theta_{T\text{-}8}) \end{aligned} $$

$$ \mathbf{Y} = 0 $$

$$ \begin{aligned} \mathbf{Z} = \; & 39{,}17 \cdot \sin(\Theta_{T\text{-}8}) \cdot \cos(\Theta_{T\text{-}9}) \cdot \cos(\Theta_{T\text{-}10}) + 39{,}17 \cdot \cos(\Theta_{T\text{-}8}) \cdot \sin(\Theta_{T\text{-}9}) \cdot \cos(\Theta_{T\text{-}10}) - \\ & 39{,}17 \cdot \cos(\Theta_{T\text{-}8}) \cdot \cos(\Theta_{T\text{-}9}) \cdot \sin(\Theta_{T\text{-}10}) - 39{,}17 \cdot \sin(\Theta_{T\text{-}8}) \cdot \sin(\Theta_{T\text{-}9}) \cdot \sin(\Theta_{T\text{-}10}) + \\ & 66 \cdot \sin(\Theta_{T\text{-}8}) \cdot \cos(\Theta_{T\text{-}9}) + 66 \cdot \cos(\Theta_{T\text{-}8}) \cdot \sin(\Theta_{T\text{-}9}) + 76{,}66 \cdot \sin(\Theta_{T\text{-}8}) \end{aligned} $$

In the following, the two basic trigonometric transformations are used:

$$ \begin{aligned} \sin(x_1 \pm x_2) &= \sin(x_1) \cdot \cos(x_2) \pm \cos(x_1) \cdot \sin(x_2) \\ \cos(x_1 \pm x_2) &= \cos(x_1) \cdot \cos(x_2) \mp \sin(x_1) \cdot \sin(x_2) \end{aligned} $$

This leeds to the following simplification:

$$ \mathbf{X} = 39{,}17 \cdot \cos(\Theta_{T\text{-}8} + \Theta_{T\text{-}9} + \Theta_{T\text{-}10}) + 66 \cdot \cos(\Theta_{T\text{-}8} + \Theta_{T\text{-}9}) + 76{,}66 \cdot \cos(\Theta_{T\text{-}8}) $$

$$ \mathbf{Z} = 39{,}17 \cdot \sin(\Theta_{T\text{-}8} + \Theta_{T\text{-}9} + \Theta_{T\text{-}10}) + 66 \cdot \sin(\Theta_{T\text{-}8} + \Theta_{T\text{-}9}) + 76{,}66 \cdot \sin(\Theta_{T\text{-}8}) $$

Solving for ($\sin(\Theta_{T\text{-}8} + \Theta_{T\text{-}9} + \Theta_{T\text{-}10})$) and ($\cos(\Theta_{T\text{-}8} + \Theta_{T\text{-}9} + \Theta_{T\text{-}10})$), respectively:

$$ \cos(\Theta_{T\text{-}8} + \Theta_{T\text{-}9} + \Theta_{T\text{-}10}) = \frac{X - 66 \cdot \cos(\Theta_{T\text{-}8} + \Theta_{T\text{-}9}) - 76{,}66 \cdot \cos(\Theta_{T\text{-}8})}{39{,}17} $$

$$ \sin(\Theta_{T\text{-}8} + \Theta_{T\text{-}9} + \Theta_{T\text{-}10}) = \frac{Z - 66 \cdot \sin(\Theta_{T\text{-}8} + \Theta_{T\text{-}9}) - 76{,}66 \cdot \sin(\Theta_{T\text{-}8})}{39{,}17} $$

Applying the fact that $\tan(x) = \frac{\sin(x)}{\cos(x)}$:

$$ \tan(\Theta_{T\text{-}8} + \Theta_{T\text{-}9} + \Theta_{T\text{-}10}) = \frac{Z - 66 \cdot \sin(\Theta_{T\text{-}8} + \Theta_{T\text{-}9}) - 76{,}66 \cdot \sin(\Theta_{T\text{-}8})}{X - 66 \cdot \cos(\Theta_{T\text{-}8} + \Theta_{T\text{-}9}) - 76{,}66 \cdot \cos(\Theta_{T\text{-}8})} $$

The angle $\Theta_{T\text{-}10}$ of the thumb then can be calculated:

$$\Theta_{T\text{-}10} = \operatorname{atan}\left(\frac{(Z - 66 \cdot \sin(\Theta_{T\text{-}8} + \Theta_{T\text{-}9}) - 76{,}66 \cdot \sin(\Theta_{T\text{-}8}))}{(X - 66 \cdot \cos(\Theta_{T\text{-}8} + \Theta_{T\text{-}9}) - 76{,}66 \cdot \cos(\Theta_{T\text{-}8}))}\right) - \Theta_{T\text{-}8} - \Theta_{T\text{-}9}$$

The angle $\Theta_{T\text{-}10}$ can be solved provided that the angle $\Theta_{T\text{-}9}$ is known. The angle $\Theta_{T\text{-}9}$ will be determined in the following subsection.

For the solution of the angle $\Theta_{T\text{-}9}$ of the thumb the trigonometric relation is used:

$$\cos(x)^2 + \sin(x)^2 = 1$$

therefore: $\cos(\Theta_{T\text{-}8} + \Theta_{T\text{-}9} + \Theta_{T\text{-}10})^2 + \sin(\Theta_{T\text{-}8} + \Theta_{T\text{-}9} + \Theta_{T\text{-}10})^2 = 1$

Using the previously obtained relation for $\cos(\Theta_{T\text{-}8} + \Theta_{T\text{-}9} + \Theta_{T\text{-}10})$ and $\sin(\Theta_{T\text{-}8} + \Theta_{T\text{-}9} + \Theta_{T\text{-}10})$:

$$\left(\frac{X - 66 \cdot \cos(\Theta_{T\text{-}8} + \Theta_{T\text{-}9}) - 76{,}66 \cdot \cos(\Theta_{T\text{-}8})}{39{,}17}\right)^2 + \left(\frac{Z - 66 \cdot \sin(\Theta_{T\text{-}8} + \Theta_{T\text{-}9}) - 76{,}66 \cdot \sin(\Theta_{T\text{-}8})}{39{,}17}\right)^2 = 1$$

This ecuation does not contain the angle $\Theta_{T\text{-}10}$ and the angle $\Theta_{T\text{-}8}$ is already predetermined. Solving this Ecuation leeds to the following result:

$$a_1 = 2 \cdot 66 \cdot 76{,}66 \cdot \sin(\Theta_{M\text{-}8}) - 2 \cdot 66 \cdot Z$$

$$a_2 = 2 \cdot 66 \cdot 76{,}66 \cdot \cos(\Theta_{M\text{-}8}) - 2 \cdot 66 \cdot X$$

$$a = \sqrt{a_1^2 + a_1^2}$$

$$k = -Z^2 - X^2 + 2 \cdot 76{,}66 \cdot (X \cdot \cos(\Theta_{M\text{-}8}) + Z \cdot \sin(\Theta_{M\text{-}8})) - 8698{,}4667$$

$$\Theta_{T\text{-}9} = \begin{cases} \pi - \operatorname{asin}\left(\frac{k}{a}\right) - \Theta_{T\text{-}8} - \operatorname{atan}\left(\frac{a_2}{a_1}\right) & \text{if } a1 \ge 0 \text{and } a2 \ge 0 \\ -\pi - \operatorname{asin}\left(-\frac{k}{a}\right) - \Theta_{T\text{-}8} - \operatorname{atan}\left(\frac{a_2}{a_1}\right) & \text{if } a1 \ge 0 \text{and } a2 < 0 \\ -\operatorname{asin}\left(\frac{k}{a}\right) - \Theta_{T\text{-}8} - \operatorname{atan}\left(\frac{a_2}{a_1}\right) & \text{if } a1 < 0 \end{cases}$$

4.1.6 Determination of the orientation of thumb

The determination of the configuration of the hand so far calculates the middle finger with the palm and independently of its orientation the thumb. The only connection between the two parts is the point $P_{T\text{-}C}$. It is necessary to determine the orientation of the thumb in relation to the palm in order to decide whether the found configuration is a good configuration and also to decide whether it is a possible configuration with respect to the range of the two articulations $\Theta_{T\text{-}7}$ and $\Theta_{T\text{-}8}$.

Calculation of the angle $\Theta_{T\text{-}7}$:

The angle $\Theta_{T\text{-}7}$ is determined by two planes:

- The first plane is formed by the vector that points from the point $P_{T\text{-}C}$ to the point $P_{T\text{-}S}$ with $\Theta_{T\text{-}7}$ to $\Theta_{T\text{-}11}$ set to zero (which is the zero-configuration of the thumb) and by the vector that points from the point $P_{T\text{-}C}$ to the point $P_{T\text{-}S}$ with $\Theta_{T\text{-}7}$ to $\Theta_{T\text{-}11}$ set to zero, except $\Theta_{T\text{-}8}$ set to 90º. With the two vectors the normal vector of the plane can be determined.

- The second plane is formed by the vector that points from the point $P_{T\text{-}C}$ to the point $P_{T\text{-}S}$ with $\Theta_{T\text{-}7}$ to $\Theta_{T\text{-}11}$ set to zero and by the vector that points from the point $P_{T\text{-}C}$ to the point $P_{T\text{-}S}$ using the values calculated for the configuration of the thumb (see Subsection-4.1.1).

The calculation of the angle $\Theta_{T\text{-}7}$ then is the angle between the normal vectors of the two planes. Depending if the point $P_{T\text{-}S}$ is located on the left or the right handside of the first plane, the resulting angle $\Theta_{T\text{-}7}$ will have negative or positive sign. This problem can be solved by calculating the distance (see Subsection-6.3.4) of the point $P_{T\text{-}S}$ from the first plane without using the absolute values. If the sign of the result is positive, the sign of the angle $\Theta_{T\text{-}7}$ has to be inverted.

Calculation of the angle $\Theta_{T\text{-}8}$:

The angle $\Theta_{T\text{-}8}$ is determined by the angle between the following two vectors: The first vector is the vector that points from $P_{T\text{-}C}$ to $P_{T\text{-}S}$, the second vector points from $P_{T\text{-}C}$ to the point $P_{T\text{-}S}$ with $\Theta_{T\text{-}7}$ to $\Theta_{T\text{-}11}$ set to the calculated values, except $\Theta_{T\text{-}8}$ which is set to 0º. The applied method uses the following two distances to determine the angle between these two vectors:

- The first is the distance between $P_{T\text{-}C}$ and $P_{T\text{-}S}$. This is the distance from the point $P_{T\text{-}S}$ to the rotating axis of the angle $\Theta_{T\text{-}8}$.

- The second distance is defined by the distance of the point P_{T-S} from the plane that is formed by the following two vectors: The vector that is pointing in the direction of the rotating axis of the angle Θ_{T-8} and the vector that points from P_{T-C} to the point P_{T-S} with Θ_{T-7} to Θ_{T-11} set to the calculated values, except Θ_{T-8} which is set to 0º.

The angle Θ_{T-8} then is calculated by calculating the arcsine of the quotient of the two distances. Since the calculated distances never have a negative sign it is unavoidable to check whether the angle has to be inverted.

4.1.7 Determination of fingertips of middle finger and thumb

With the determination of the position and the orientation of the palm, the middle finger and the thumb together it is possible to calculate the two remaining articulations of each finger which describe the sphere in the fingertips (Θ_{T-11}, Θ_{T-12}, Θ_{M-11}, Θ_{M-12}). The determination for the middle finger and the thumb is basically the same, therefore in the following subsections the angle $\Theta_{Finger-11}$ is used to specify the angles Θ_{T-11} and Θ_{M-11}, furthermore the angle $\Theta_{Finger-12}$ is used to specify the angles Θ_{T-12} and Θ_{M-12}.

For the calculation of the angles $\Theta_{Finger-11}$, $\Theta_{Finger-12}$ it is necessary to obtain the orientation of the last two falanges of the corresponding finger and relate its position to the position and the orientation of the contact point with the object.

Calculation of the angle $\Theta_{Finger-11}$:
For the determination of the angle $\Theta_{Finger-11}$ it is necessary to define the plane that is formed by the two vectors that form the last two falanges of the finger. With the distance of the contact point of the object to this defined plane and the distance to the the Vector that is the axis of rotation of the angle $\Theta_{Finger-11}$ the angle can be calculated. The angle $\Theta_{Finger-11}$ then is the arc-sine of the quotient of the two distances.

The distances are defined as positive values and the arc-sine returns values between -90º and +90º, therefore it is unavoidable to check whether the resulting angle is in the range +90º to +270º.

Calculation of the angle $\Theta_{Finger-12}$:
For the determination of the angle $\Theta_{Finger-12}$ it is necessary to define two planes. First the plane that is formed by the two vectors that form the last two falanges of the finger. With the cross-product of these two vectors the normal vector of the plane can be obtained.

The second plane then is formed by this normal vector and the distance of the contact point of the object to this defined plane and the distance to the the Vector that is the axis of rotation of the angle $\Theta_{Finger-11}$ the angle can be calculated. The angle $\Theta_{Finger-11}$ then is the arc-sine of the quotient of the two distances.

The distances are defined as positive values and the arc-sine returns values between -90º and +90º, therefore it is unavoidable to check whether the resulting angle is in the range +90º to +270º and in that case correct the angles.

4.1.8 Position of the hand-base

The calculation of the position of the palm and the middle finger permits to determine the location of the base of the hand. With this information it will be possible to calculate the position of the base of the index finger and the ring finger.

The relative position of the point $P_{T\text{-}C}$ within the hand is static (see Subsection-4.1.2) and also the absolute position of $P_{T\text{-}C}$ in the workspace is known (see Subsection-4.1.3). With the angles Θ_4 and Θ_5 (see Subsection-4.1.4) then it is possible to determine the position of the hand-base.

$$P_{H-B} = P_{T-C} + \left({}^{5}_{4}T \cdot {}^{6}_{5}T \cdot (-P_{T-C}) \right)$$

First the hand is positioned in the world-reference with $\Theta_4 = 0$ and $\Theta_5 = 0$. The vector pointing from $P_{T\text{-}C}$ to the world-reference will then be rotated with the transformation matrices for Θ_5 and for Θ_4. Adding $P_{T\text{-}C}$ results in the linear transformation of the hand, the result $P_{H\text{-}B}$ then gives the position of the hand-base in the workspace.

4.1.9 Index finger and ring finger

With the determination of the middle finger, the palm and the thumb it is possible to calculate the configuration of the index finger and the ring finger. The position of the base of the last two fingers is calculated with their forward cinematic and in Subsection-4.1.1 the center of the spheres are determined.

4.1.9.1 Determination of articulations of falanges of index and ring finger

Once the configuration of the middle finger and the thumb are determined the angles $\Theta_{I\text{-}8}$, $\Theta_{I\text{-}9}$ and $\Theta_{I\text{-}10}$ for the articulations of the index finger and the ring finger can be determined in a similar way as already described in Subsection-4.1.4 where the middle finger is positioned with the difference that the distance of interest the distance between the center of sphere to the base of the corresponding finger is.

4.1.9.2 Determination of fingertips of index finger and ring finger

The angles (Θ_{I-11}, Θ_{I-12}, Θ_{R-11}, Θ_{R-12}) of the articulations of the index finger and the ring finger are determined in the same way as already explained for the middle finger and the thumb in Subsection-4.1.7.

4.1.10 Calculation of the quality of the arrangement

The calculation of the overall quality of the arrangement ($\text{Quality}_{\text{Arrangement}}$) is divided into three parts. The first part($\text{Quality}_{\text{Hand}}$) is the quality of the hand only considering the articulations of the fingers without the fingertip. The second part ($\text{Quality}_{\text{Fingertips}}$) is the quality of the articulations in the fingertips and the third part ($\text{Quality}_{\text{Object-height}}$) is determined by both the height that the actual configuration of the hand would permit an object to occupy and the desired height. The overall quality-function is calculated as follows:

$$\text{Quality}_{\text{Arrantgement}} = \text{Quality}_{\text{Hand}} \cdot \text{Quality}_{\text{Fingertips}} \cdot \text{Quality}_{\text{Object-height}}$$

In all of the individual quality-functions an exponential part can be found that ensures that the search converges towards a possible configuration which permits the desired height.

4.1.10.1 Quality of the articulations of the hand (without fingertips)

In order to be able to decide whether the configuration of the hand is suitable and also to allow an improvement, starting from the initial position for P_{T-C} found with the heuristic method (see Subsection-4.1.3), it is necessary to apply a target function which returns the quality of the configuration. The target function proposed in this work favors solutions in which the angles of the articulations remain close to the middle of their range and penalizes angles which are out of their range and therefore would result in an impossible configuration of the hand.

First of all a quality is calculated for each of the articulations depending on its range. The quality-function returns a value between 0 and 1.

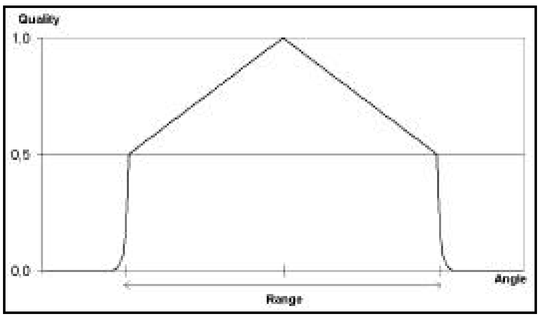


Figure 8. **Quality-function for the articulations of the hand.**

Fig.8 shows that the quality-function is divided into two parts. The linear part is applied for angles that can be found within their range, returning values between 0.5 and 1. The non-linear part is applied for angles which are out of their range, the function converges exponentially towards zero. The reason for using this exponetially decreasing function is to ensure that the search is to penalize angles that are out of range and to permit the search algorithm to converge towards a possible solution.

The equation for the quality-function applied for the articulations is the following:

$$\text{Quality}_{\text{Hand}} = \begin{cases} 1 - \left|\dfrac{\Theta - \Theta_{\text{Mid-Range}}}{\Theta_{\text{Sum-Range}}}\right| & \text{if } \Theta \in \text{Range} \\[2ex] \dfrac{0.5}{0.5 + \left|\dfrac{\Theta - \Theta_{\text{Mid-Range}}}{\Theta_{\text{Sum-Range}}}\right|} & \text{if } \Theta \notin \text{Range} \end{cases}$$

with Θ the angle of the articulation, $\Theta_{\text{Mid-Range}}$ the value in the middle of the range and $\Theta_{\text{Sum-Range}}$ the maximal value minus the lower value of the range.

The exponential part of the quality-function ensures that the search applied later on to the point $P_{T\text{-}C}$, in order to improve the configuration, results in a possible configuration of the hand.

4.1.10.2 Quality of the articulations of the fingertips

The quality of the articulations of the fingertips can be calculated in the same way as for the rest of the articulations, but because a little change in the articulations of the fingers could result in a very big change of the articulations of the fingertips, the quality function was modified slightly. Instead of the linear function for angles within the range, its square is used which result in a configuration of the hand that has the last falange in a more orthogonal configuration with respect to the surface of the object.

4.1.10.3 Quality of the height of the object

Without the search that will be applied it would be sufficient to use a binary variable indicating that the configuration of the hand permits the desired height of the object or not. Due to the search it is necessary to apply a quality-function for the height which has the value 1 if the desired height can be reached and is exponentially decreasing towards 0 if not. Fig.9 shows the quality-function for the height.

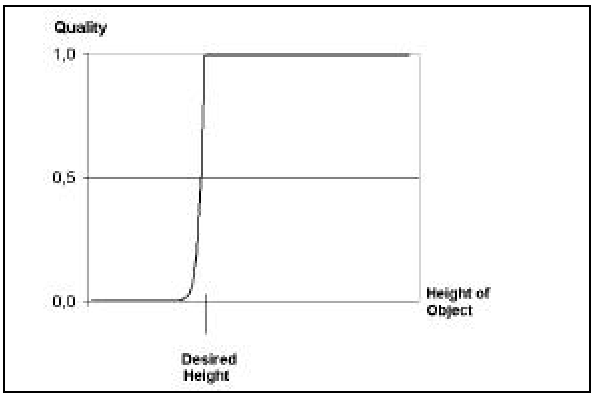


Figure 9. **Quality-function for the height that the configuration of the hand permits an object to occupy.**

The function then is the following:

$$\text{Quality}_{\text{Height of object}} = \begin{cases} 1 & \text{if } \Theta \in \text{ Range} \\ \dfrac{0.5}{0.5 + \left|\dfrac{\Theta - \Theta_{\text{Mid-Range}}}{\Theta_{\text{Sum-Range}}}\right|} & \text{if } \Theta \notin \text{ Range} \end{cases}$$

The rapidly increasing part of the function ensures that the search converges towards a solution that permits at least the desired height.

4.1.11 Search for improvement of the configuration

Once found the initial configuration of the hand, it must be ensured that the fingers are able to reach their final destination and make contact with the object. The used heuristic method (explained in Subsection-4.1.3) does not necessarily result in a possible configuration because of the range of the angles. Specially the angle $\Theta_{I\text{-}7}$ usually is out of its range for this initial configuration, this results from the fact that its range is very small.

Therefore it is necessary to apply a guided search that compares the found solution with the surrounding solutions in order to improve the position of the point $P_{T\text{-}C}$ and its resulting configuration. The search finally will find a configuration that is really applicable and does allow the hand to grasp the object with its desired height and width.

The search is done using the quality-functions explained in Subsection-4.1.10 with the objective to find a configuration of the hand that results in angles of the articulations that stay close to the middle of their range.

The search compares the quality of the initial configuration of the hand placed in the initial position of $P_{T\text{-}C}$ with solutions found by displacing the point $P_{T\text{-}C}$ around its initial position. The best solution is selected serves as a new starting point of the search. This is incremental search is performed until the best solution is found. In order to decrease the calculation time the search uses two different increments. The bigger value ensures that the search converges rapidly towards the final position of $P_{T\text{-}C}$ and only searches in the X- and Z-direction, where the smaller value is used for the precise search in all of the three directions.

The search with the fine increments results in the best global solution, taking in account that it is a solution found with discrete steps. In the case that the final solution would not be precise enough and would need improvement, it can be imagined to use a Newton-Iteration in order to find a solution with more precision within the final two increments.

4.1.12 Output file in VRML-format

The simulation written in C^{++} stores the result for the obtained configuration of the hand into a file using the language VRML. The utilized objects are basic shapes that can be programed with VRML, like spheres, cylinders, cubes and polygons (defined by a number of points). Using the two basic methods “translate” and “rotate” it is possible to construct the hand consisting of the palm and the four fingers, each finger consisting of three phalanges.

The found solution for the hand then determines the value of rotation of the individual articulations of the fingers and the final position of the wrist determines the translation of the hand. The found solution then shows the hand grasping the object that is defined by its width and height.

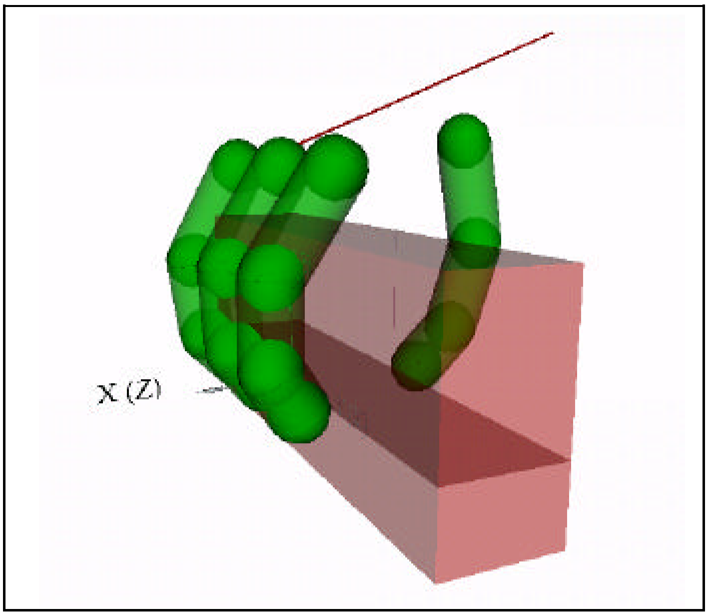


Figure 10. **Visualisation of simulation-results with VRML.**

PART 5

Results

The simulation written in C^{++} is used to calculate the solutions for the possible range of width and height of the object to be grasped. For the width as well as for the height a range of 0 mm to 200 mm was choosen with increments of 1 mm. This leads to a total of 40000 calculations. The obtained results show the possible range objects that the hand can grasp using the proposed method. In this section the necessary qualities are presented which indicate the quality of the respective configurations.

5.1 Desired height of object

The inputs used in the simulation are the width and the height of the parallelepiped. In principle position of the object also can be changed which will result in a the same configuration of the hand, shifted by the same vector of displacement as the object.

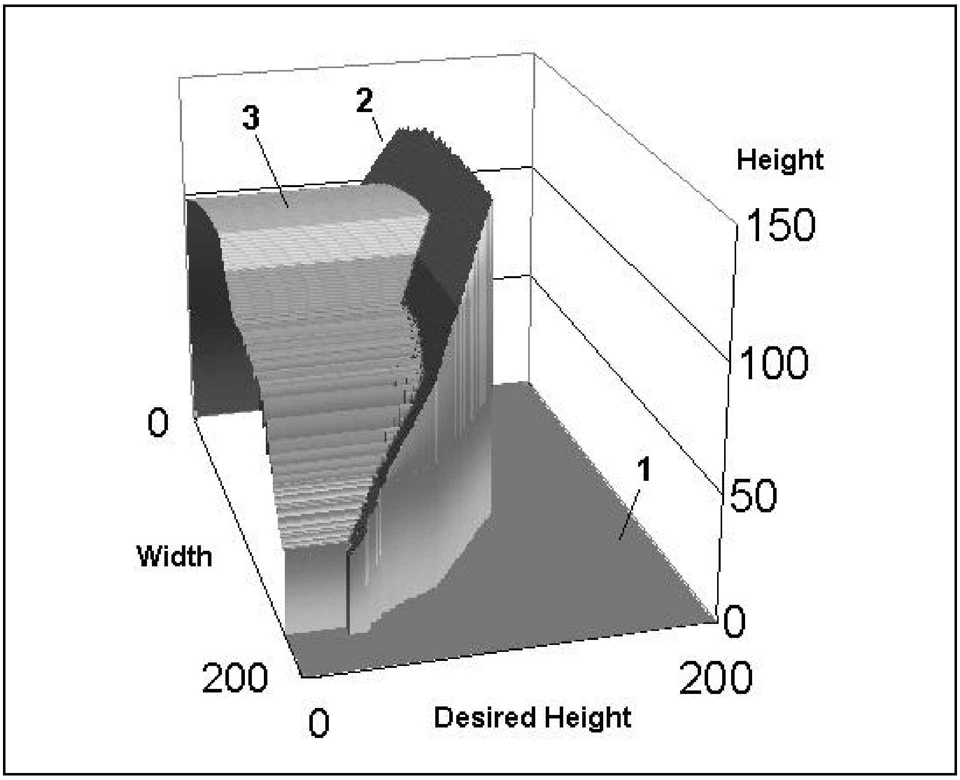


Figure 11. **Possible height of object. (1) No possible solution found. (2) The Configuration fulfills the desired height. (3) The found configuration permits a bigger height for the object.**

The graph in Fig.11 shows the result of the simulation with respect to the possible height and width of an object to be grasped. The results can be divided into three parts: The first part (1) where no possible configuration is found for the desired values. In other words the object to be grasped is too big. The second part (2) indicates the solutions which fulfill the desired grasp of the object. A plane with an angle of 45º is formed due to the fact that for a desired height the obtained height has the same value. In this part the applied search was dominated by fulfilling the desired height. Finally the third part (3) displays the solutions that result in a possible configuration of the hand but would permit a bigger height for the object. It can be seen that in this part the heigth maintains constant for input values that have the same desired width for the object. This results in the fact that a configuration is found that is best for a certain width.

The same three parts can be observed in the graph showing the quality of the respective configurations (see Fig.12).

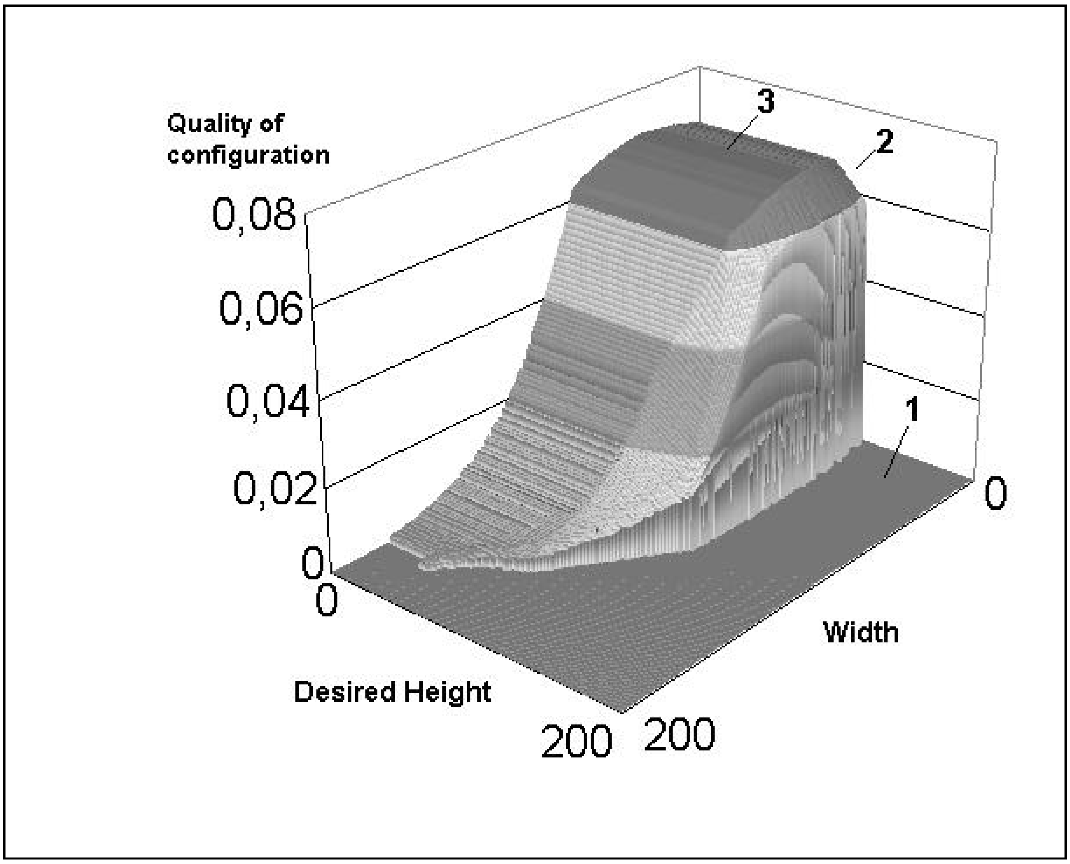


Figure 12. **Quality of the configuration of the hand. (1) No possible solution found. (2) The search is dominated by fulfills the desired height. (3) The search finding an optimum configuration for the hand, dominated by the desired width of the object.**

The first part (1) shows that no solution is found for the desired objects. The second part (2) is dominated by fulfilling the desired height of the object and the quality of the configuration decreases rapidly while the hand approaches more unusual configurations.

As already explained, the third part (3) shows that the configuration for a desired width of the object does not result in a change of the configuration of the hand and permits the object to intrude more into the hand without any further change.

The values for the quality of the configuration, which indicates its quality, do not reach the value “1”, even though the quality of an individual articulation can have the value “1”. This can be explained by the fact that the quality is calculated considering both the articulations of the hand and the angles that are found in the virtual articulatins considered in the sphere of the fingertips depending on the orientation of the surface of the object.

PART 6 *Appendix*

6.1 Tables of the Representation of Denavit-Hartenberg

i	α_{i-1}	a_{i-1} /mm	d_i /mm	Range	Offset of Θ_i
4	90º	0	0	0º...360º	
5	-90º	0	0	0º...360º	
6	90º	0	343		-44,56º
7	14,11º	7,56	-203,32	33,44º...57,44º	
8	90º	0	0	0º...90º	
9	0º	76,66	0	0º...90º (100º)	
10	0º	66	0	0º...90º (100º)	
11	0º	39,17	0	20º...135º	
12	-90º	0	0	-45º...45º	
13	0º	20	0	0º	

Table 3. **Representation of "Denavit-Hartenberg" for thumb.**

i	α_{i-1}	a_{i-1} /mm	d_i /mm	Range	Offset of Θ_i
4	90º	0	0	0º...360º	
5	-90º	0	0	0º...360º	
6	90º	0	276,55	-180º...360º	+90º
7	90º	67	9,5	78º...102º	
8	90º	0	0	0º...90º	
9	0º	76,66	0	0º...90º (100º)	
10	0º	66	0	0º...90º (100º)	
11	0º	33,62	0	20º...135º	
12	-90º	0	0	-45º...45º	
13	0º	20	0	0º	

Table 4. **Representation of "Denavit-Hartenberg" for index finger.**

i	α_{i-1}	**a_{i-1}** /mm	**d_i** /mm	**Range**	**Offset of Θ_i**
4	90º	0	0	0º...360º	
5	-90º	0	0	0º...360º	
6	90º	0	276,55	-180º...360º	+90º
7	90º	0	9,5	78º...102º	
8	90º	0	0	0º...90º	
9	0º	76,66	0	0º...90º (100º)	
10	0º	66	0	0º...90º (100º)	
11	0º	33,62	0	20º...135º	
12	-90º	0	0	-45º...45º	
13	0º	20	0	0º	

Table 5. **Representation of "Denavit-Hartenberg" for middle finger.**

i	α_{i-1}	**a_{i-1}** /mm	**d_i** /mm	**Range**	**Offset of Θ_i**
4	90º	0	0	0º...360º	
5	-90º	0	0	0º...360º	
6	90º	0	276,55	-180º...360º	+90º
7	90º	-67	9,5	78º...102º	
8	90º	0	0	0º...90º	
9	0º	76,66	0	0º...90º (100º)	
10	0º	66	0	0º...90º (100º)	
11	0º	33,62	0	20º...135º	
12	-90º	0	0	-45º...45º	
13	0º	20	0	0º	

Table 6. **Representation of "Denavit-Hartenberg" for ring finger.**

6.2 Schematic of hand

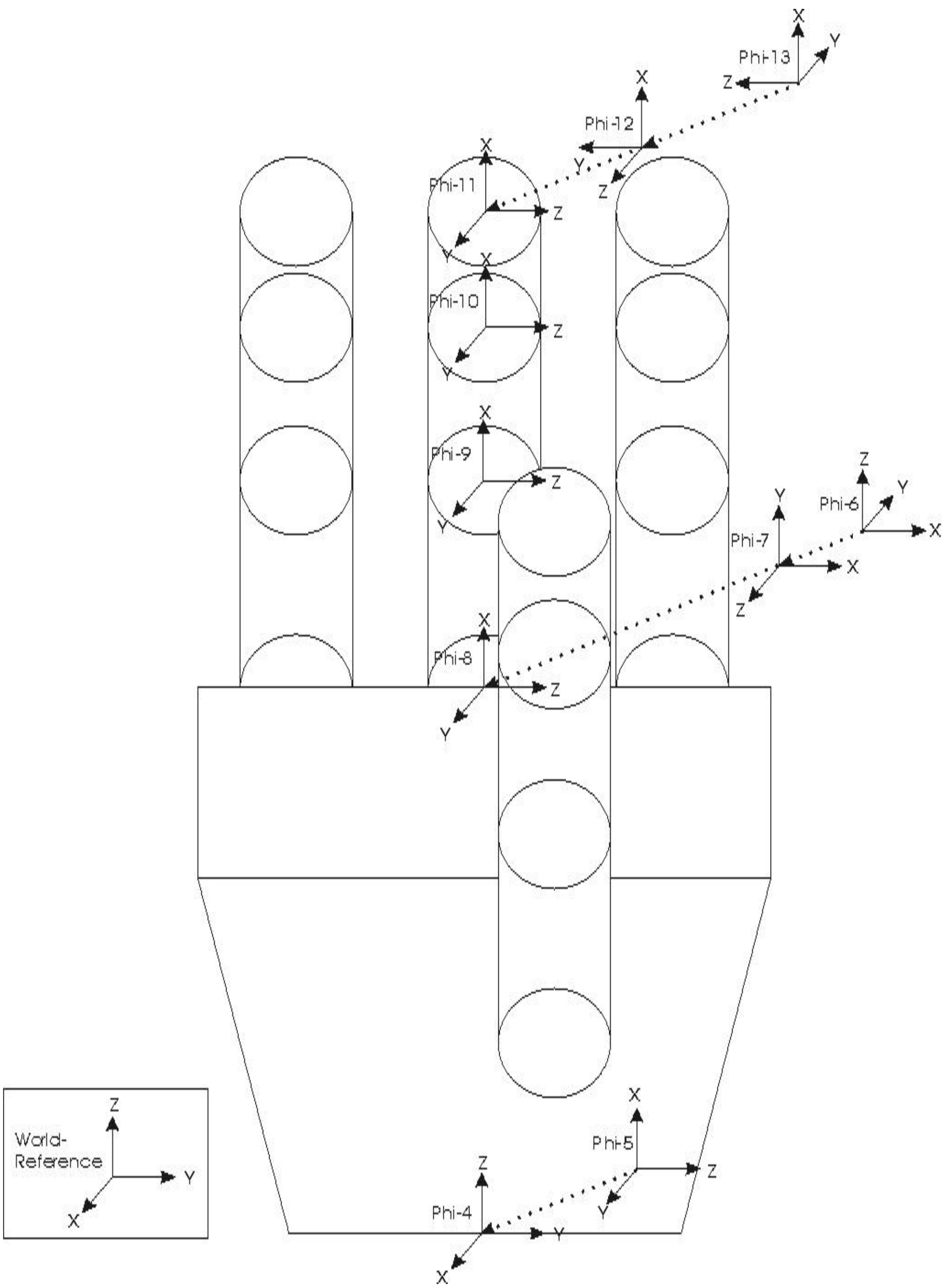


Figure 13. Schematic of mechanical hand showing the coordinate systems for the representation of Denavit-Hartenberg for the middle finger.

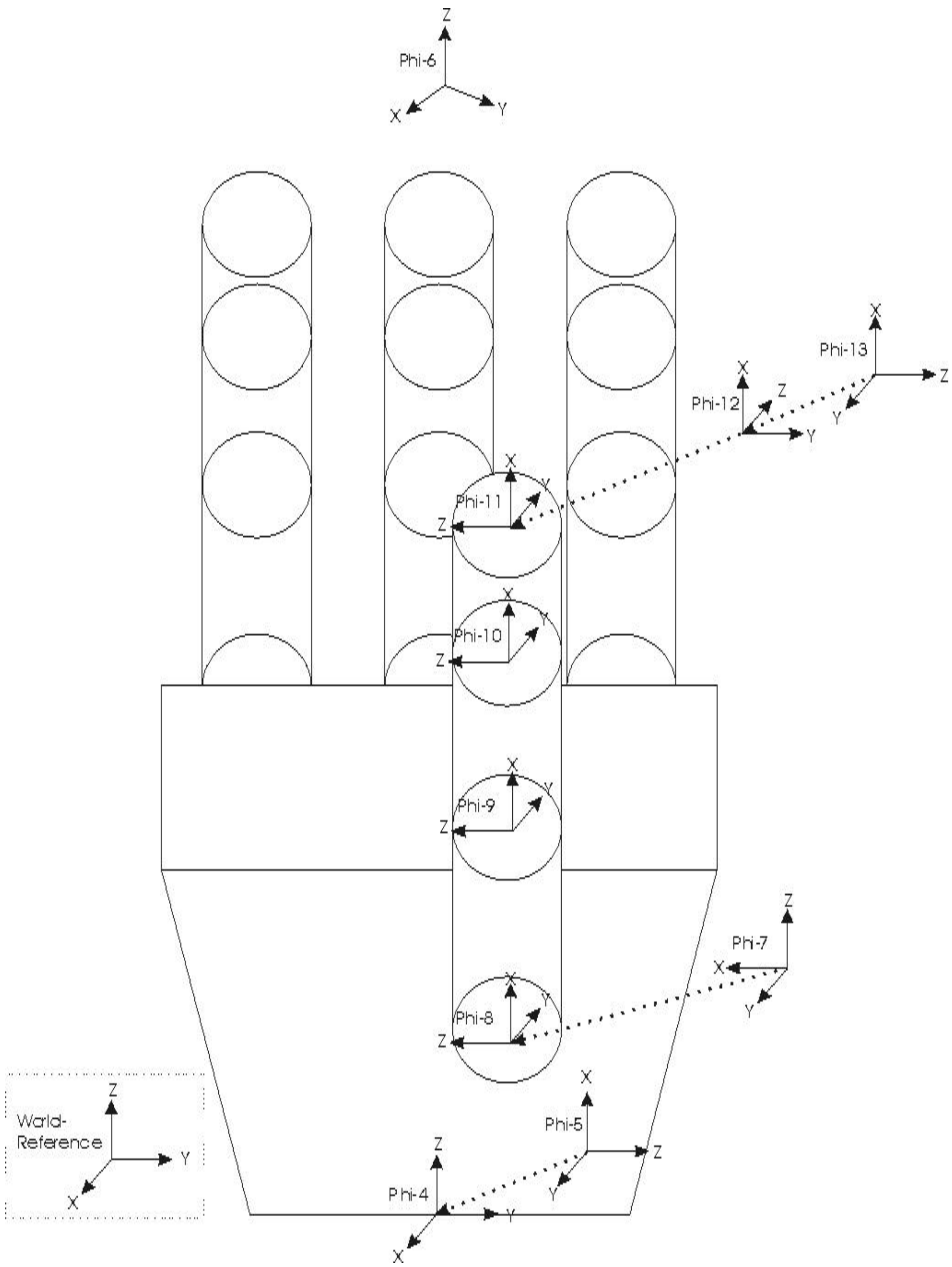


Figure 14. **Schematic of mechanical hand showing the coordinate systems for the representation of Denavit-Hartenberg for the Thumb (with Θ_8 set to 90º).**

6.3 Mathematical basis of Vector Analysis

6.3.1 Scalar product

The scalar product of two vectors $\vec{a}$ and $\vec{b}$ is defined as follows:

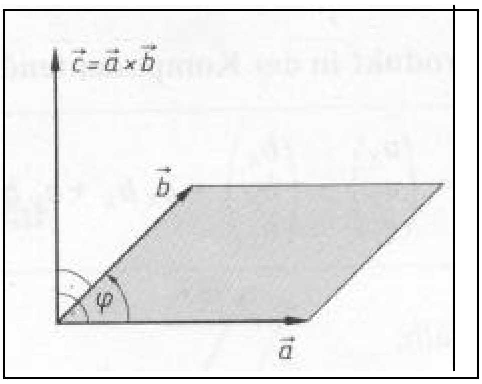
$$\vec{a} \cdot \vec{b} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \cdot \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = a_x b_x + a_y b_y + a_z b_z$$

Two vectors are perpendicular if their scalar product is equal a zero.

6.3.2 Cross product

The cross product of two vectors $\vec{a}$ and $\vec{b}$ is defined as follows:

$$\vec{c} = \vec{a} \times \vec{b} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \times \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = \begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix}$$

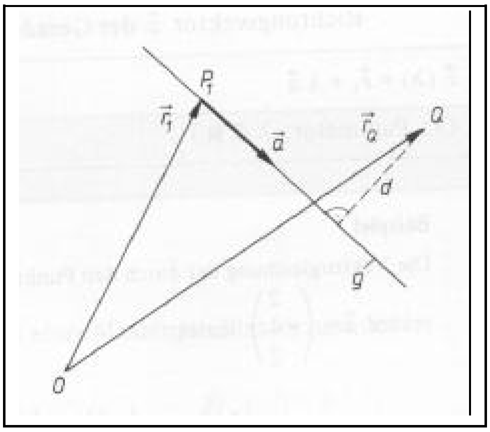


The vector $\vec{c}$ is perpendicular to $\vec{a}$ and $\vec{b}$. The orientation of a plane which is defined by two vectors $\vec{a}$ and $\vec{b}$ also can be defined by its normal vector $\vec{n}$ which can be calculated with the cross product of the two vectors.

6.3.3 Distance Point-Vector

A straight line is defined by a vector $\vec{a}$ indicating its direction and a vector $\vec{r}_1$ pointing from the origin towards a point on the line. The distance d then is the distance between the straight line and a point Q with its corresponding origin-vector $\vec{r}_Q$:

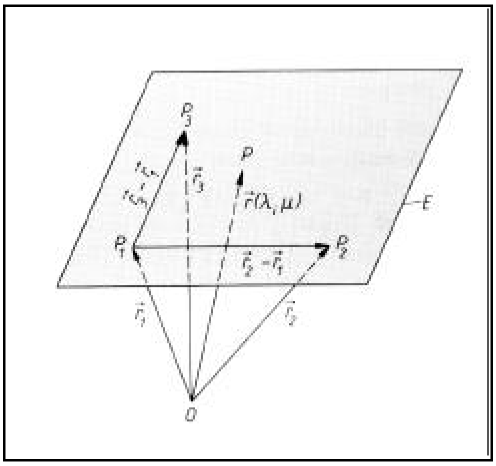
$$d = \frac{\left|\vec{a} \times (\vec{r}_Q - \vec{r}_1)\right|}{\left|\vec{a}\right|}$$



6.3.4 Distance Point-Plane

A plane is defined by its normal vector $\vec{n}$ and a vector $\vec{r}_1$ pointing from the origin towards a point in the plane. The distance d then is the distance between the plane and a point Q with its corresponding origin-vector $\vec{r}_Q$:

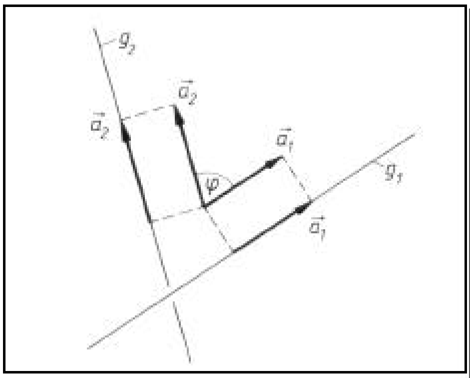
$$d = \frac{\left|\vec{n} \cdot (\vec{r}_Q - \vec{r}_1)\right|}{\left|\vec{n}\right|}$$



6.3.5 Angle Vector-Vector

The angle φ between two vectors $\vec{a}$ and $\vec{b}$ is defined by the following equation:

$$\varphi = \operatorname{acos}\left(\frac{\left|\vec{a}_1 \cdot \vec{a}_2\right|}{\left|\vec{a}_1\right| \cdot \left|\vec{a}_2\right|}\right)$$

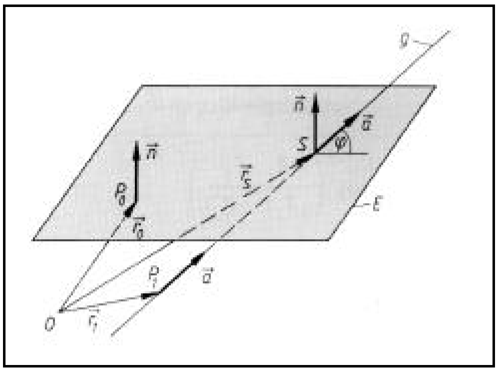


The angel φ also exists in the case that the two vectors do not intersect in the space.

6.3.6 Angle and Intersection Vector-Plane

The angle φ between a straight line (with $\vec{a}$ and $\vec{r}_1$) and a plane (with $\vec{n}$ and $\vec{r}_0$) is defined by the following equation:

$$\varphi = \operatorname{asin}\left(\frac{|\vec{n} \cdot \vec{a}|}{|\vec{n}| \cdot |\vec{a}|}\right)$$



The vector to the point of intersection $\vec{r}_S$ of a straight line (with $\vec{a}$ and $\vec{r}_1$) and a plane (with $\vec{n}$ and $\vec{r}_0$) is defined by the following equation:

$$\vec{r}_S = \vec{r}_1 + \frac{\vec{n} \cdot (\vec{r}_0 - \vec{r}_1)}{\vec{n} \cdot \vec{a}}\vec{a}$$

Condition: The plane and the straight line only intersect if $(\vec{n} \cdot \vec{a} \neq 0)$.

PART 7

Bibliography and used Software

Bibliography

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Software

In terms of the utilized software, the analytical calculations were obtained from Maple 7.00, the schematics were drawn in CroelDRAW9.0, the simulation is programed in Microsoft® VisualC++ 6.0, the 3D-Illustration of the results was done in Microsoft® Excel 2000 and the document was written in Adobe® Framemaker® 6.0.